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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

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PHOTOMETRY OF $H\beta$ IN THE CHROMOSPHERIC SPECTRUM OUTSIDE OF ECLIPSE

By PHILIP C. KEENAN

ABSTRACT

Photometric measurements on spectrograms taken at the limb of the sun outside of eclipse were carried out to give preliminary contours of $H\beta$ at various heights in the chromosphere. The errors entailed by the observational integration of the false light superimposed upon the true chromospheric spectrum by scattering and scintillation in the earth's atmosphere prevent the deduced contours and intensities from being very exact. Observations at time of eclipse will probably be necessary to furnish precise results.

However, the width of the line can be determined more accurately and is found to be strikingly constant throughout the lower 2000 km of the chromosphere. In this same region the total intensity also varies but little.

The measurements were used in connection with Unsöld's equation for intensities within emission lines in the chromosphere to obtain approximate average values for the coefficient of scattering and the density of hydrogen atoms in the layers above 300 km. The computations gave 12×10^{-10} for s_0 and 3000 atoms per cubic centimeter for N.

Photographic study of the spectrum of the chromosphere has been based chiefly upon observations of the flash spectrum near second and third contacts of total solar eclipses, and although such observations have been assiduously undertaken at nearly every eclipse since the first successful registration of the spectrum of the chromosphere on a photographic plate by A. Fowler and by W. Shackleton in 1893, it is only within the last few years that a start has been made in the direction of photometric measurements.

Such photometry as has been attempted has consisted chiefly of estimates of relative intensities of the lines, or, as in the work of the Dutch observers at the eclipse of 1927,[1] of determinations of the total

[1] A. Pannekoek and M. G. J. Minnaert, *Verhandelingen der Koninklijke Akademie van Wetenschappen te Amsterdam*, **13**, No. 5, 1928; **14**, No. 2, 1930.

energy emitted within lines. A third important factor, the variation of intensity with height, has been investigated to some extent, the first quantitative work having been done by E. F. Carpenter on the plates taken with a moving camera by the Lick Observatory expedition to the Spanish eclipse of 1905.[1] On the other hand, detailed study of the structure of individual lines has in all cases been precluded not only by the relatively small dispersion employed but also by the method of forming the spectrum, since the slitless prismatic camera usually adapted for work of this sort gives a series of images of the chromospheric crescent, somewhat blurred by the finite widths of the frequency intervals within which emission takes place. Such spectra are of some use in estimating total intensities, but in connection with the recent theoretical work on chromospheric emission it is important to have knowledge of the contour of a line as a function of the perpendicular distance of the line of sight from the base of the chromosphere. For this purpose spectrograms taken with a narrow slit and a dispersion of at least 10 A to the millimeter are essential, and such material has up to now been available only from the few observations made on the chromosphere outside of eclipse.

The successful photographs taken at Mount Wilson with the 60-foot tower telescope by G. E. Hale and W. S. Adams[2] first brought out the richness of the data obtainable from observations made in full daylight, although both H. Deslandres and Hale had secured plates showing some of the chromospheric lines as early as 1891, and the latter had afterward employed the Kenwood spectroscope at the Yerkes Observatory for the same purpose. In a survey of the region $\lambda\lambda$ 4800–6600 Adams and Miss Burwell[3] were able to tabulate more bright lines than were visible on the eclipse plates taken by S. A. Mitchell, and felt that they were reaching layers closer to the photosphere, remarking: "It seems probable that double reversal is a universal characteristic of all lines in the chromospheric spectrum at a low level."

However, it must be recognized that unsteadiness and scattering in the earth's atmosphere offer the greatest difficulties to the securing

[1] *Lick Observatory Bulletins*, **12**, No. 384, 183, 1927.

[2] *Astrophysical Journal*, **30**, 222, 1909.

[3] *Ibid.*, **41**, 116, 1915.

of spectrograms suitable for photometry. In this connection it is interesting to note the views of Mitchell, written after he had attempted to use the same instruments at Mount Wilson that had been employed by Adams. He states:

> In the short time at his disposal for making observations, the writer came to the conclusion that for photographing the flash spectrum without an eclipse the best possible conditions are necessary, or in other words with seeing and steadiness of quality 10 on a scale where 10 represents the best possible. If therefore one wishes to obtain such photographs without an eclipse he must be continually on the alert to catch the few moments of perfect seeing when they come; and then after taking hundreds or possibly thousands of photographs for this purpose a photograph may finally be secured which will be of quality equal or better than that of Adams obtained twenty years ago. In fact, the importance of excellent seeing in attempting to secure photographs of the flash spectrum, no matter what method is employed, cannot possibly be overemphasized.[1]

Under any given conditions of observation, the errors due to atmospheric effects are greatly diminished as the image of the sun thrown on the slit of the spectrograph is made larger and brighter. On a large image not only can the slit be placed accurately, but the moments of best seeing can be picked out more easily than when the sun's disk is small. Furthermore, the reduction in exposure time made possible by greater intensity helps very much in diminishing scintillation, the disturbances with periods longer than the exposure being largely eliminated.

These considerations serve to stress the need for the utilization of the greatest possible optical power if any hope of success in treating the problems of the chromosphere is to be entertained. For a comparison in this respect of the equipment employed at the several observatories that have carried out chromospheric photography Table I was prepared, giving the instruments together with the diameters of the objective and of the image of the sun.

The 150-foot tower at Mount Wilson, used by A. Unsöld to obtain a few contours of $H\alpha$,[2] H,[3] and K in emission, is unequaled for the size of the image produced. However, the small objective entails long exposures, adding to the uncertainty of the heights ob-

[1] *Handbuch der Astrophysik*, **4**, 294, 1929.

[2] *Zeitschrift für Physik*, **59**, 353, 1929.

[3] *Astrophysical Journal*, **69**, 209, 1929.

served. Unsöld estimated that his contours for H and K corresponded to altitudes of 2500 and 5600 km, but in the case of $H\alpha$ he was able to say only that the observation referred to the middle of the chromosphere.

More recently he has determined the mean contour of λ 5876 (D_3) of helium with the spectrograph of the Einstein tower at Potsdam. The same difficulty was encountered there, as the exposures were ten minutes in length, leading him to say: "Bei dieser Art der Aufnahme ist zwar die Höhe über dem Sonnenrand nicht genau definiert—sie war bei uns schätzungsweise 5000 bis 10,000 km. . . . ."[1]

The Arcetri solar tower has been employed for measurements of wave-length of the chromospheric lines (in addition to the regular

TABLE I

INSTRUMENTS SUITABLE FOR CHROMOSPHERIC PHOTOGRAPHY

Observatory	Instrument	Objective	Diameter of Image
Arcetri	Solar tower	30 cm	17 cm
Meudon	Horizontal telescope	25	3.6
Mount Wilson	60-ft. tower	30	18
Mount Wilson	150-ft. tower	30	43
Potsdam	Einstein tower	60	13
Yerkes	40-in. telescope	102	18

spectroheliographic program) rather than for photometry. The positions of the components of $H\alpha$, $H\beta$, and calcium H have been determined accurately by G. Abetti and B. Nováková,[2] who appear not to have attempted any quantitative work on the intensities.

In general, then, it may be seen that the 40-inch refractor at Yerkes is among the most powerful instruments adaptable to this type of work. For this reason it seemed desirable to make use of it for measurements of the contours of some of the stronger chromospheric lines at different levels. In the present paper no attempt has been made to eliminate all the sources of photometric errors, but preliminary data for $H\beta$ are presented as a foundation for further work. It will appear that there are a few characteristics of the line that stand out as relatively independent of the many disturbing factors.

[1] *Zeitschrift für Astrophysik*, **3**, 77, 1931.

[2] *Osservazioni e memorie del R. Osservatorio Astrofisico di Arcetri*, No. 43, 23, 1926; No. 46, 27, 1927.

OBSERVATIONS

The plates were taken with the Rumford spectroheliograph, adapted for use as an ordinary spectrograph by opening the second slit. The arrangement of a plane grating, ruled with twenty thousand lines to the inch, placed in front of the two prisms normally utilized in taking spectroheliograms,[1] increases the scale to 3.5 A/mm, which, while not as great as might be desired, is nevertheless adequate for lines as broad as $H\beta$. With this dispersion it was possible at the same time to keep the exposures reasonably short, from five to fifteen seconds.

Because of the many reflecting surfaces in this optical arrangement, the elimination of instrumental scattering was not complete even with numerous diaphragms between the parts; in order to minimize it, a filter transmitting only in the blue-green was inserted between the grating and the prisms.

The exposures for the calibration of photographic density of the image in terms of relative intensity of the incident light were taken through an exactly similar filter with a tube sensitometer.[2] The two duplicate sets of calibration exposures placed on each plate were in all cases impressed just before the plate-holders were loaded for exposure to the sun. The emulsion was Eastman process, developed in formula D-11. Photometric measurement of the spectrograms was carried out with a registering microphotometer, the treatment being similar to that normally given in the work on stellar line contours here.[3] The magnification from the negative to the tracing was thirty-one times, and the width of the defining slit was varied between 0.05 and 0.20 A.

The question of resolving power can best be considered empirically by examination of the microphotometer curves, where the smoothing effects of the finite widths of the slits of the spectrograph and the microphotometer as well as of the photographic spreading will be combined. On the typical tracing shown in Figure 1 the two emission components of $H\beta$ are completely separated down to the base

[1] Hale, *Publications of the Yerkes Observatory*, **3**, 1, 1903.

[2] For description of the sensitometer see P. C. Keenan, *Astrophysical Journal*, **73**, 118, 1931.

[3] C. T. Elvey, *ibid.*, **68**, 145, 1928.

of the line, so the artificial contour produced by the instrument for an infinitely narrow line of approximately the same intensity can be safely considered as having less than half of the total width of the observed line. Thus, lines one-third of an angstrom apart should be distinctly separated. Inasmuch as the two principal components in the fine structure of $H\beta$ are nearly a tenth of an angstrom (0.08 A)

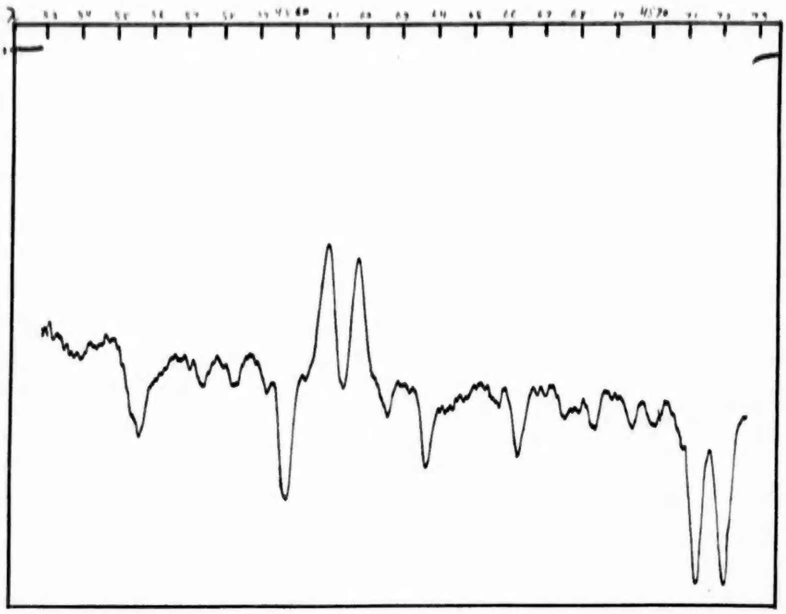


FIG. 1.—Microphotometer tracing of $H\beta$ at the base of the chromosphere, from plate 96. The two short lines near the top correspond to complete opacity. The scale of wave-lengths has been approximated as constant for the short interval shown.

apart,[1] the details of the broadened chromospheric line may be expected to be blurred by about this amount; hence a slightly greater resolving power might be advantageous, but little would be gained by increasing it very much.

The exposures were made with the slit of the spectrograph placed tangentially to the sun's limb. As may be seen from the reproductions in Plate XIII, this arrangement magnifies the vertical scale so that measurements can be made at much more definitely determined

[1] W. V. Houston, *ibid.*, **64**, 81, 1926.

PLATE XIII

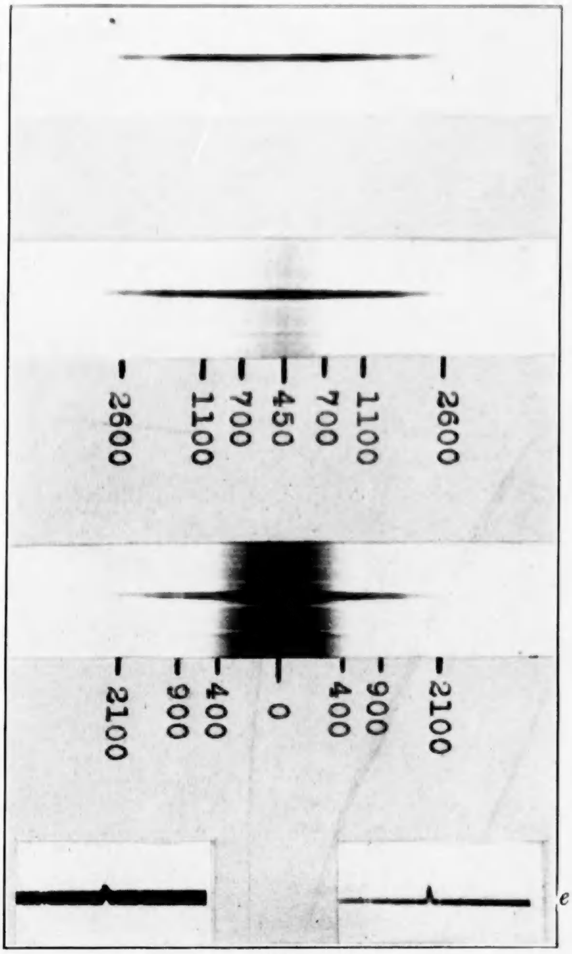
a

b

c

d

e



$H\beta$

The scales give the height in kilometers at which measurements were made. The enlargement is 2.7 times the original.

a) High level (pl. 48)
b) Medium level (pl. 25)
c) Low level (pl. 25)
d) Base of small prominence (pl. 47)
e) Same as *d*, except slightly different position

metrically placed with respect to the center, taking care to avoid obviously distorted portions of the line, the contour at each height except that corresponding to the center of the exposure was obtained from two sections of the limb separated by some thousands of kilometers. With this procedure local disturbances tend to neutralize one another.

THE SCALE OF HEIGHTS

The determination of the relative heights as a function of distance measured along the spectral line from the center is a simple matter of geometry, though the computation is lengthened by the necessity of correcting for the variation in the size of the sun's image through the year and for the curvature of the slit of the spectrograph. However, the establishment of the zero point of the scale—the height of the slit at its lowest point—offers greater difficulties and involves the question of the definition of the base of the chromosphere. In order to arrive at a practical solution of the problem it was assumed that the transition from the lower chromosphere, or reversing layer, to the photosphere is abrupt. Then the upper limit of the photosphere was taken as the level at which the brightness falls off most rapidly on exposures strong enough to show the lines of the middle chromosphere. It is at once evident that this definition makes the heights somewhat dependent on the density and contrast of the plate. However, this defect is not serious for the material under discussion, since the range in times of exposure is small. Furthermore, an independent check is available in the width of the absorption line $H\beta$. This is well known to increase toward the limb, but as soon as the emission components become considerable, they cause it to narrow again, so that the position of maximum width should also occur close to the true photospheric limb. The maximum is not very sharply defined, but at least it sets a limit to the errors of the first method, and the two criteria give results in satisfactory agreement.

The method adopted here seems justified by the general consistency of the variation of contours with height for the different exposures measured. Some degree of arbitrariness in setting up the scale of altitudes is unavoidable, and this procedure seems to be as objective as any other that might be followed.

In putting it into practice, the exposures on any one plate were made of equal length and arranged in a series, those at one end being taken with the slit set so that a centimeter or so of its length was definitely within the photosphere, while the other extreme was represented by exposures embracing only the higher chromospheric levels. On the images showing the spectrum of the disk the height of the center of the slit was obtained directly by measuring the width of the photosphere included, the edges being defined as above. In the case of those taken entirely above the disk it was necessary to measure the maximum intensity of the continuous spectrum, adopting the height corresponding to the same intensity on the exposures already calibrated. For intermediate exposures both methods were utilized, so that checks were available on the uniformity of the scale. In general, the relative heights from each plate are probably accurate within a few hundred kilometers. For a single exposure the uncertainty may be reduced to less than a hundred kilometers, but it is necessarily much more when the results from different plates are combined.

As a final check on the absolute heights, comparison may be made with the values given by Mitchell[1] from eclipse observations. The maximum elevation of $H\beta$ as observed here is only about 4000 km, whereas Mitchell obtained 8500 km. This discrepancy is not so serious when it is considered that the faint outer portions of the line are lost in observations made without eclipse. An example of this is the height of the H line of calcium as measured directly by C. E. St. John[2] with a radial slit. He found 5000 km for the maximum, as compared with Mitchell's 14,000 km. On the other hand, the lines of medium level agree excellently in the two scales. Thus the height of λ 4854.82 of *Y–Fe*, visible below $H\beta$ in Plate XIII, agrees well with the value of 500 km given by Mitchell. The difference between the results for strong and weak lines is probably due to the fact that the latter fall off in brightness much more suddenly at their upper limits.

CORRECTIONS FOR SCINTILLATION AND SCATTERED LIGHT

Figure *c* of Plate XIII shows the strong continuous spectrum that is photographed along with the emission lines even at considerable

[1] *Astrophysical Journal*, **71**, 1, 1930. [2] *Ibid.*, **32**, 36, 1910.

elevations. This illustration was made from the best plate of the series; on those taken under ordinary conditions the extent affected is much greater, and the fainter chromospheric lines are completely obscured.

It is certain that most of the light of this continuous spectrum is due to scattering from the sky and within the spectrograph and to superposition of the disk spectrum for part of the exposure by the scintillation of the sun's image. Although our assumption of a perfectly sharp edge to the photosphere is not strictly true, eclipse observations prove that the region of transition must be small, the continuous spectrum disappearing within a few hundred kilometers at most.

Of the two sources of false light, scintillation is by far the more effective. Microphotometer curves of exposures taken with a radial slit and most of the disk cut off by a screen show that the total scattered light is negligible for the moderate exposures employed, except when the sky is appreciably hazy. If an attempt were made to reach higher levels by long exposures, this factor would become important, but on nearly all the exposures concerned here scintillation alone can be considered as the source of trouble.

The problem of evaluating the correction for scintillation cannot be solved from the data at present available. The false contour at any definite height is given by the integral of the contributions from the adjacent levels on both sides, weighted according to the length of time during which the light from each level will be thrown upon the slit. The weighting factor can be represented by a probability curve,[1] but the contributing intensities are given by the unknown function of the height that is one of the objects of investigation. The rate of variation of this function from level to level is too rapid and its general form too uncertain to justify a mathematical approximation until precise measures made during total eclipses give the true curve for at least one line.

Under these circumstances the safest course is to assume a maximum value for the correction, considering that the observed line contour is compounded from the true emission line and an absorp-

[1] O. Heckmann and H. Siedentopf, *Veröffentlichungen der Universitäts-Sternwarte zu Göttingen*, Heft 8, 1929.

tion line having the shape taken by $H\beta$ just inside the disk, reduced in depth in the ratio of the intensity of the continuous spectrum at the given height to that at the limb.[1]

All the mean contours were corrected in this manner. In the figures reproduced here the upper solid curve represents the observed distribution of intensity across the line, the assumed absorption line is below, and the corrected contour is given by the broken curve.

The original and the final curve furnish two extremes between which the true contour probably lies. The presence of any residual absorption spectrum in the chromospheric light itself, the superposition of the emission line from points above that measured, and the natural tendency of the instruments of observation to smooth any sharp peaks, all work in the opposite direction to the assumed error; hence the final contour is almost certainly overcorrected by an uncertain amount.

INTENSITIES AND CONTOURS OF THE LINE AT DIFFERENT HEIGHTS

All the measures corresponding to the same height on each plate were averaged to form mean contours, the line being treated as symmetrical. There is, as is well known,[2] a consistent asymmetry in the sense that the violet-emission component is stronger and broader than the other, but the difference is in nearly all cases so slight that it scarcely shows on the microphotometer tracings. Plate 45 forms the one exception; on it the asymmetry is strongly marked in the lower levels. This point will bear further investigation.

The data available for analysis comprise contours at twenty-six separate levels; nearly all of them averaged from two or more microphotometer tracings. In some cases material from several plates is available for a given altitude.

At first glance the discrepancies seem more striking than the agreements. Figure 2 illustrates a typical case. In this and in Figure 3 the scale of intensities is such that the continuous spectrum at 0 km is represented by 100. Plates 17 and 39 gave results sufficiently similar to justify averaging, but on plate 96, taken under better conditions, the line is so much stronger that the central absorption

[1] This suggestion is due to Dr. Elvey.

[2] Abetti and Nováková, *op. cit.*

component no longer vanishes after subtraction of the intensity of the continuous spectrum. In some cases differences of this sort undoubtedly arise from real changes on the sun, but the greater share

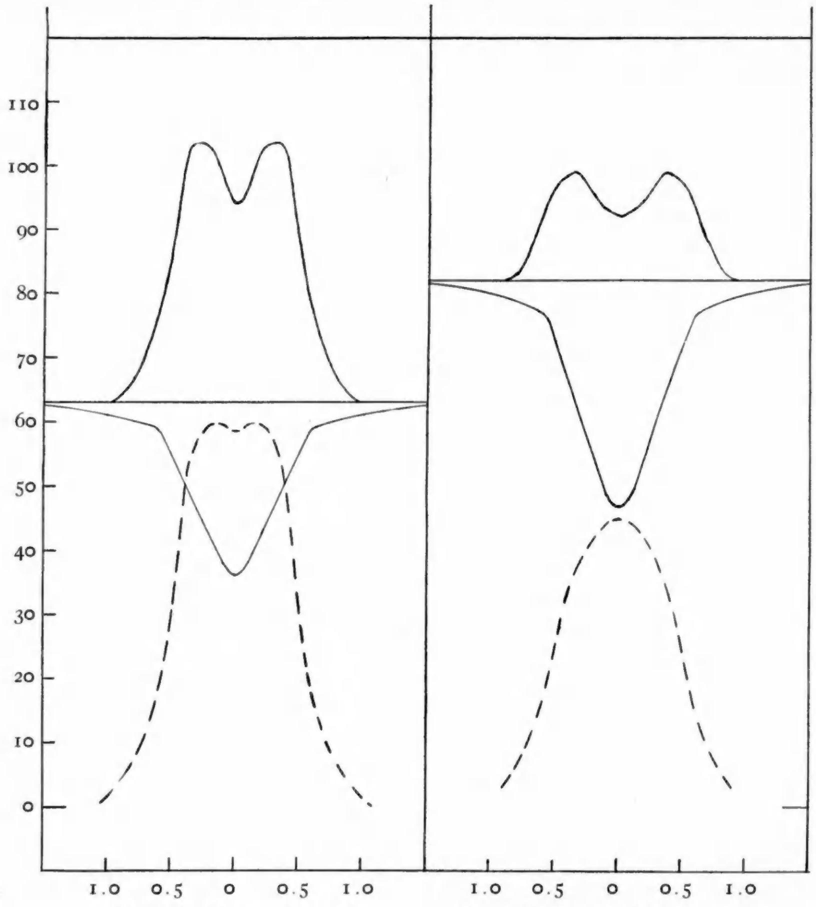


Plate 96. Height=630 km — Plates 17 and 39. Height=610 km

FIG. 2.—Contours of $H\beta$ near 600 km as obtained from different plates

originate in our atmosphere. For this reason the total observed intensities referred to the continuous spectrum mean very little.

On the other hand, the variation of the contours and the relative intensities of the lines with height has more significance, and can be determined, at least approximately, from the data at hand. The cor-

rected contours will be drawn upon chiefly in the discussion which follows.

In Figure 3 the changes in the form of the line are shown for two of the best plates. The three heights represented for each plate are

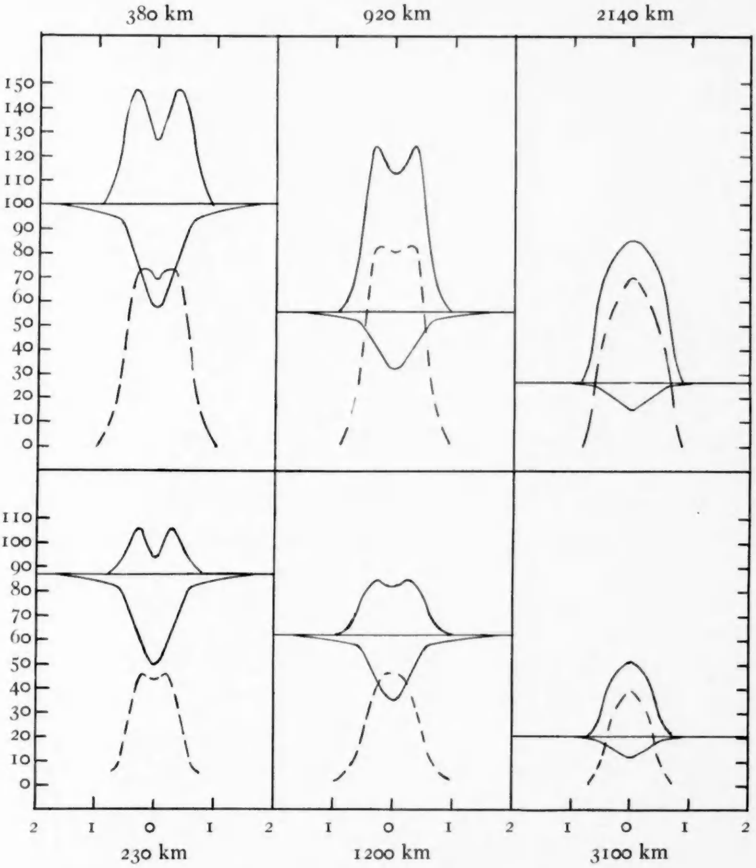


FIG. 3.—Variation of the contour of $H\beta$ with height. The upper set of curves is taken from plate 25, the lower from plate 45.

sufficient to show the progressive development from a single emission line to one having two distinct maxima with an absorption line between them, in passing to lower levels. This well-known characteristic of most of the stronger chromospheric lines is present without important modifications on all the plates. The height at which the

absorption component appears to vanish, measured directly on the negatives, varies from 1100 to 2000 km, though plate 55 is the only one giving a value of less than 1500 km. Since none of the corrected contours shows the double line above 1100 km, it may be concluded that the absorption component actually due to the chromosphere itself has an upper limit between 1000 and 2000 km above the photosphere.

Further information on this point can be gained by plotting the corrected maximum intensities as a function of altitude. This has

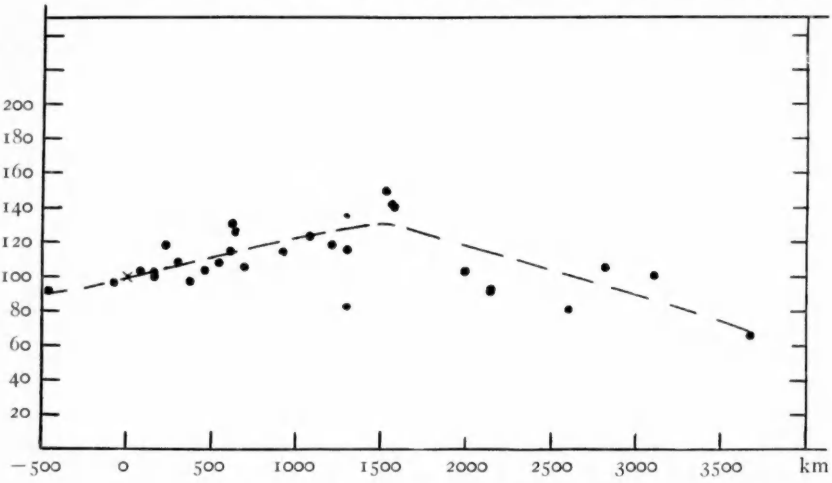


FIG. 4.—Corrected maximum intensities as a function of height. The intensities are expressed in percentages of the value for the same plate at 0 km.

been done in Figure 4, where the intensities from the different plates have been made homogeneous by expressing them in terms of the intensity at 0 km on the same plate. Thus the points all tend to coincide near this value, the dispersion becoming greater with increasing heights. The plotted intensities show clearly a systematic distribution, indicated roughly by the dotted line. It is worth noting that the only point which falls far below this mean curve is one from the poorest of the plates included. The curve is characterized by a uniform rise for the first 1500 km, at which height the intensity is about 25 per cent greater than that at the base of the chromosphere; farther out its course becomes less certain, but it appears to go down with a slightly greater slope than that of the ascending part.

It is possible, of course, that the apparent rise in the lower range of heights is due to systematic errors for which the greater uncertainty of the correcting factor in this region is responsible, but the agreement between the position of maximum intensity and the height of disappearance of the absorption component suggests that the increase is real. If this is true, the phenomenon is independent evidence in favor of the conclusion that self-reversal in the chromosphere produces the absorption component. With increasing self-reversal with greater depth, the top of the emission line would be cut off to a greater and greater extent as the photosphere is approached, thus accounting for the observations.

One other possible explanation must be admitted. Disregarding self-reversal entirely, the emission line itself may have a maximum at a definite elevation above the base of the chromosphere if below this height the conditions of excitation are such that the number of hydrogen atoms in the second quantum state is increasing fast enough to more than counterbalance the decreasing density.[1]

Much additional data will probably be necessary to settle this question; the more so since it is likely that the observed effect is at least exaggerated by the atmospheric sources of error.

WIDTH OF THE EMISSION LINE

On each mean corrected contour the width of the line was measured at two points: (1) at an intensity five units above the base and (2) at half the maximum intensity. The first gives a measure of the width at the base, but is seriously influenced by slight errors in the intensities. A more definitive measure is given by the second, for here the sides of the line are steeper. In Table III this quantity is designated as the "Half-Value Width," although the term here does not have quite the same meaning as in its normal use in connection with absorption lines, where it signifies the width at the position where the coefficient of absorption has half its maximum value.

The base width exhibits considerable fluctuations, with a general tendency to decrease toward the outer chromosphere. On the contrary, the half-value width stays remarkably constant between -500 and 2200 km. Within this range the average value is 1.02 ± 0.013 A.

[1] The possibility of an explanation of this sort was pointed out to me by Dr. Otto Struve.

From the discussion of resolving power it is apparent that a line of this size is only slightly affected by the instrumental contour. The measured breadth is therefore characteristic of the true line. Its absolute value would be modified appreciably by changes in the

TABLE III

WIDTHS OF $H\beta$ IN EMISSION

Plate	Height	Base Width	Half-Value Width
39	−460 km	1.7 A	0.95 A
96	− 80	1.85	1.10
17	0	1.4	0.95
45	0	1.8	1.15
50	0	1.8	0.95
39	80	1.6	0.95
50	160	1.6	0.95
55	160	1.5	0.90
45	230	1.5	0.95
25	310	1.6	1.10
25	380	1.8	1.15
25	460	1.85	1.15
55	540	1.6	0.95
39, 17	610	1.7	1.05
96	630	1.7	1.05
25	690	1.7	1.05
25	920	1.65	1.05
25	1070	1.5	0.85
45	1200	1.6	1.00
17	1300	1.2	0.95
50	1300	1.75	1.25
55	1300	1.55	1.00
39	1520	1.6	0.85
50	1550	1.8	1.20
96	1570	1.6	1.05
17	1990	1.5	1.00
17	2140	1.45	0.90
25	2140	1.75	1.15
25	2600	1.25	0.80
39	2800	1.4	0.90
45	3100	1.2	0.85
17	3700	1.15	0.85

correcting factor, as a glance at the contours in Figures 2 and 3 will show, but its uniformity throughout the whole of the lower chromosphere is a striking result of the present investigation and one that is only slightly affected by the variable errors arising from the conditions of observation. Contrast effects make it difficult to see this phenomenon visually, but exposures at moderately high levels, such as Figures *a* and *b* of Plate XIII, show it rather well.

On account of the correlation to be expected between breadth and total emission, the observed constancy in width suggests that the intensities of the line should not change by a very large factor in the same range of heights—a conclusion which has been confirmed by Figure 4.

APPLICATION OF UNSÖLD'S THEORY OF THE CHROMOSPHERE

The only comprehensive theory of the emission spectrum of the chromosphere was derived by Unsöld[1] in 1927 and amplified by him in the succeeding two years.[2] On the assumption of radiative equilibrium he developed from E. A. Milne's equations for the emissivity as a function of optical depth an expression giving the intensity at every point in the line in terms of the coefficient of scattering (s) and the perpendicular distance (y) from the line of sight to the outer edge of the chromosphere. His equation in its general form is

$$I=\frac{J_0}{1+\frac{2\omega}{1+\omega}\bar{s}H}\left\{1-\frac{3}{2Rs}+3\sqrt{\frac{y}{2R}}-e^{-2s\sqrt{2Ry}}\left(1-\frac{3}{2Rs}-3\sqrt{\frac{y}{2R}}\right)\right\}, \quad (1)$$

where ω is $\sqrt{k/(k+s)}$; J_0 is the intensity of outward directed photospheric radiation at the limb; k is the coefficient of absorption; H is the total height of the chromosphere; $\bar{s}$ is the mean value of the coefficient of scattering for all heights; and R is the radius of the sun.

Solution for the quantities outside the braces requires observed values of the intensities within the line in terms of the brightness of the continuous spectrum at some definite point on the disk. Since measures of this sort are not yet available, a precise test of the accuracy of the theoretical contours is not now possible. Qualitatively, however, as Unsöld has already shown in the papers cited, it fits the observations well, and since no serious theoretical objections have been raised against the equation, it offers a promising means for the determination of the coefficient of scattering. Better observed contours will be necessary before this can be done with any considerable

[1] *Zeitschrift für Physik*, **46**, 782, 1927.

[2] *Astrophysical Journal*, **69**, 209, 1929.

degree of accuracy, but a test computation with the data presented in this paper may not be without interest.

For such a rough solution the portion outside the braces may be written

$$\frac{J_0}{1+sH}\,,$$

which is Unsöld's earlier approximate form, and all the intensities expressed in terms of the constant J_0.

Let the height at which the contour is to be computed be 300 km. From the several good measures made near this level (see Fig. 3) the separation of the peaks of the emission components is close to 0.5 A. The problem now is to find the s_0, the value of s at the center of the line, that will give rise to this distance between the components, assuming with Unsöld that the variation of s with wave-length is due to turbulent motion of the gases in the chromosphere. Taking the distribution of velocities as Maxwellian, Unsöld arrived at the expression

$$s=\frac{\sqrt{\pi}\,e^2\lambda_0^2 Nf}{mc^2\Delta\lambda_0}\,e^{-\left(\frac{\Delta\lambda}{\Delta\lambda_0}\right)^2}\,, \tag{2}$$

where e is the charge on an electron; m is the mass of an electron; c is the velocity of light; λ_0 is the wave-length at the center of the line; N is the number of atoms per cubic centimeter; f is the oscillator strength; $\Delta\lambda_0$ is the Doppler shift corresponding to the most probable velocity; and $\Delta\lambda$ is the distance from the observed point to the center of the line.

This may be written

$$s=s_0e^{-\left(\frac{\Delta\lambda}{\Delta\lambda_0}\right)^2}\,. \tag{3}$$

For both calcium and helium Unsöld has found the mean velocity of turbulence to be about 15 km/sec. Taking this as the velocity of the hydrogen atoms also—and the similarity in the motions of the three gases in prominences offers some support for this procedure—$\Delta\lambda_0$ becomes 0.2 A.

The value to be ascribed to y depends upon the total thickness of the chromosphere. Unsöld made use of the heights given by Mitchell (12,000 km for $H\alpha$), but for comparison with observations made outside of eclipse there would seem to be almost equal justification for taking the maximum value (about 5000 km) obtained under such conditions. The difference affects the fit of the curves strongly, as a given height would correspond to a contour, under the assumption of a small total thickness, that would belong to a much greater height if the larger value for the thickness were taken. A middle course was adopted here by setting H equal to 8500 km, the height attributed to $H\beta$ in Mitchell's list. Then y became 8.2×10^8 cm.

It was found that the positions of the two maxima of intensity were quite sensitive to changes in s_0. The observed $\Delta\lambda_{max}$ of 0.25 A was obtained with $s_0 = 12 \times 10^{-10}$. By reducing s_0 to about 6×10^{-10}, $\Delta\lambda_{max}$ becomes 0.20 A, so that the given value is definite within a factor of about 2. Some additional computations carried through for $H = 12{,}000$ km gave a separation of 0.30 A, from which it follows that the possible error due to this source is roughly the same as the original uncertainty of the determination. The computed half-value width of $H\beta$ comes out about 0.8 A, and the ratio of the intensity in the central absorption component to the maximum nearly 0.65. The differences between these and the observed quantities may be due to instrumental spreading of the line, or may call for modification of some of the constants of the equations, such as the velocity of turbulence. This last factor is particularly uncertain in the case of hydrogen, for which the thermal velocity at an absolute temperature of 4800° is 10.7 km/sec. as compared to 1.7 km/sec. for the atoms of Ca^+. Since the assumed turbulent motion of the calcium atoms must be practically independent of thermal agitation, it might be reasonable to take the velocity of each hydrogen atom as the vector sum of the components due to this turbulence and to thermal agitation rather than to ascribe the same motion to both H and Ca^+. Then the resultant velocity would be the root mean square of the two components, or 18 km/sec., which would probably improve slightly the agreement between the theoretical and observed curves.

It is interesting to note that the value of s_0 found here is inter-

mediate between those determined for H and K, 7.5×10^{-10} and 15×10^{-10}, respectively, by Unsöld.

The s_0 in question is the mean value for the coefficient of scattering per cubic centimeter of gas in all the layers above the given one, 300 km. By putting the observed s_0 back into equation (2), we have a means for determining N, the average number of hydrogen atoms in the second energy-level throughout this range of heights.

In carrying out the computation, the numerical value of f for $H\beta$ was taken as 0.12.

N was then easily found to be 21,000 atoms per centimeter. Multiplying by y, the total number of atoms in a column 1 cm^2 in cross-section above 300 km comes out 24×10^{12} for a chromospheric height of 12,000 km, or 17×10^{12} if the total thickness is taken as 8500 km.

In view of the roughness of these estimates, which cannot presume to give more than the order of magnitude of the quantities involved, the results appear quite reasonable and serve to indicate the importance of securing more accurate contours permitting a series of determinations of s_0 and N at different levels, which would give directly the detailed distribution in density.

Further observations on $H\beta$ and the other lines of hydrogen made outside of eclipse should lead to closer approximations, but it is highly desirable that attempts be made to photograph some of the lines at the coming total eclipse with a size of image and dispersion adequate for the determination of true contours. As several observers have pointed out,[1] observations of this sort can be made most advantageously near the edge of the path of totality, and even where the eclipse is only partial, but has a magnitude of 0.9 or more, it should be possible to obtain valuable data. H. F. Newall took advantage of the partial phases of the eclipse of April 17, 1912, to photograph the flash spectrum as early as thirty-five minutes before totality,[2] but the failure of the Cambridge observers to obtain more than a few bright lines with the same instruments at the eclipse of 1921 suggests the difficulties of the observations.

[1] E. B. Frost, *Popular Astronomy*, **26**, 297, 1918.

[2] *Monthly Notices of the Royal Astronomical Society*, **72**, 536, 1912.

I am indebted to Drs. Struve and Elvey for help and suggestions too numerous for specific acknowledgment. I wish also to thank Messrs. C. D. Higgs and H. F. Schwede for assistance in adjusting the spectrograph.

YERKES OBSERVATORY
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Since this paper was prepared D. H. Menzel has published the results of his detailed study of the photographs of the flash spectrum taken with a moving plate by the expeditions of the Lick Observatory to the eclipses of 1898, 1900, 1905, and 1908.[1] His data on the intensities of the lines, including some measures of contours, are supplemented by an extensive theoretical discussion and provide material for an interesting comparison with the results obtained above.

[1] *Publications of the Lick Observatory*, **17**, 1, 1931.

SOME RESULTS FROM A STUDY OF THE ATOMIC LINES IN THE SUN-SPOT SPECTRUM[1]

By CHARLOTTE E. MOORE

ABSTRACT

A detailed study of the atomic lines in the sun-spot spectrum has resulted in a determination of the effective temperature of sun-spots and the pressure and amount of material above them.

Intensities for 6312 lines of atomic origin in the spot spectrum have been estimated on the Rowland scale on original negatives and on prints made from them. The plates were those taken with the 150-foot tower telescope and 75-foot spectrograph for the Mount Wilson map of the spot spectrum. The third order was used from λ 3900 to λ 5400, the second order from λ 5400 to λ 6000. The spectrum of the disk appears on each side of the spot spectrum. The use of a quarter-wave plate and Nicol prism gives the lines subject to Zeeman effect a dentate appearance in the spectrum of the spot, which makes it possible to select the atomic lines. Intensities estimated from the prints agree systematically with those from the plates.

The *identifications* of lines in the solar spectrum from λ 2975 to λ 6635 and in the spot spectrum from λ 3894 to λ 6635 have been revised and extended. Lines of Lu^+ have been identified for the first time. The principal criteria employed have been laboratory wave-lengths and intensities, the multiplet relations and excitation potentials, which are now available for almost all elements except the rare earths, and the behavior of the lines in the spectrum of γ Cygni, a giant star of class F8. Nine hundred and thirty-six faint solar lines not previously identified and 21 spot lines have been recognized as faint members of multiplets not yet observed in the laboratory, but accurately predictable. Of these, 437 are due to Fe, 122 to Cr, 89 to Ni, 73 to Cr^+, 67 to Ti, 50 to Fe^+, 43 to Ti^+, 21 to Co, 20 to Zr^+, 12 to V, 9 to V^+, 7 to Sc^+, and 7 to Mn. Three hundred and ninety-six new atomic lines which appear only in the spot spectrum have been measured and 193 identified.

A *quantitative* study of the composition of the atmosphere above sun-spots has been based on the calibration of the Rowland intensity scale. This gives a quantity Y, which is the logarithm of the ratio of the number of atoms producing a line in the spot spectrum to that effective in the formation of the same line in the solar spectrum. From thermodynamic theory, $Y=Y_0+SE_x$, where Y_0 is the logarithm of the ratio of the numbers of neutral atoms above equal areas of the photospheres of spot and disk, E_x the excitation potential, and $S=5040\,(1/T-1/T')$. T and T' are the effective temperatures of disk and spot, respectively. For enhanced lines, Y_0 must be replaced by Y_1, the corresponding quantity for ionized atoms. The observed data from 6 elements give a weighted mean value of S equal to -0.190 ± 0.010. Taking $T=5740°$ K, $T'=4720°\pm40°$ K, where the probable error is a measure of the accordance of the observations. From the observed values of Y_0 and Y_1 and the Saha equation as modified by Pannekoek and Fowler, the ratio of the electron pressures P_e' (spot) and P_e (disk) can be found. Eight elements have a sufficient number of arc and spark lines present in both solar and spot spectra to be used. The weighted mean value of log (P_e'/P_e) is -0.22 ± 0.07, i.e., the electron pressure over the spot is 0.60 ± 0.10 times that over the disk.

Russell has defined the "level of ionization" as the ionization potential of an atom which would be just 50 per cent ionized under given conditions. For the spot this level is 7.0 ± 0.1 volts, as compared with 8.5 ± 0.1 for the disk. From these quantities the percentages of ionization for disk and spot were calculated for 28 elements. By comparing these percentages with Y_0 and Y_1, it was found that the amount of material per unit

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 446; continued from this *Journal*, April, 1932.

area above the spot is greater than that above the disk. The ratio of these amounts, C, was determined for 19 elements in the neutral state and for 11 in the ionized state, the result being

$$\log C = +0.23 \pm 0.03 ,$$

or

$$C = 1.70 \pm 0.11 .$$

Checks upon these results may be obtained in two ways: First, the mean percentage of ionization can be computed for both disk and spot from Russell's data for the composition of the solar atmosphere; this gives the ratio of the number of ions per square centimeter above disk and spot, which should equal the ratio of electron pressures. The calculated value of log (P'_e/P_e) is -0.17, as against -0.22 observed. Second, from Gaunt's relation that the absorption coefficient κ varies as $P_e T^{-3/2}$, it is possible to calculate the increased transparency above the spot and to determine both C and P'_e/P_e theoretically when T' is given. For $T' = 4720°$, $\log C = +0.13$ and $\log (P'_e/P_e) = -0.26$. The agreement with observation is satisfactory, in view of the approximate nature of the theory.

Adams and Russell, in determining the temperatures of stars by the method used here, introduced an empirical correction X to care for a supposed departure from thermodynamic equilibrium. If applied to the spot spectrum, this correction gives $T' = 4290°$, $\log (P'_e/P_e) = -1.09$, and $\log C = +0.31$. The first theoretical check gives log $(P'_e/P_e) = -0.11$; the second gives $\log (P'_e/P_e) = -0.38$ and $\log C = +0.19$. This discordance indicates decisively that no such empirical correction is applicable in the case of the sun-spot spectrum.

From the three constants T', P'_e/P_e, and C derived from observation, the intensities of the atomic lines in the spot spectrum may be calculated for all lines of known energy relations. These calculations have been made for a number of the most difficult cases in the whole spectrum—the lines of H, O, Si, Si^+, Mg, and Mg^+. Aside from a few lines so strong that estimates are meaningless, the average discordance of observed and calculated spot intensities is ± 1 unit of the Rowland scale. Similar calculations made for all lines in a selected region in the red show good agreement between observed and calculated values, except in the case of certain lines due to Ti, V, Y, and Zr which are very faint in the spectrum of the disk. Further study of such cases is desirable. Apart from this difficulty, the intensities of lines in the sun-spot spectrum have been quantitatively explained by physical and thermodynamical theory.

3. BEHAVIOR OF ELEMENTS

Much has already been written about the behavior of some of the elements in the sun-spot spectrum. Brief statements are given here, together with notes on the presence of certain additional elements in the sun. Since the investigation has not been confined to the spot spectrum, it has been possible to discuss the presence of several elements identified in the sun by lines which appear outside the violet and red limits of the spot region. All these results are included in the following résumé, which is compiled mostly from material in the *Revised Rowland*, collected here for convenience and completeness.

H The Rowland intensities of the first five lines of the Balmer series range from 40 to 5 in the spectrum of the disk. All are decidedly weakened in the spot spectrum. One line of the Paschen series has been observed by

H. D. Babcock at λ 10049.5. *H* is also present in compounds in both disk and spot.

He Detected by its presence in the chromosphere, where it is conspicuous. One line, λ 5875.618, is often present as an absorption line over disturbed regions of the disk.

Li The evidence of its presence in the sun rests entirely on two lines at λ 6707, which are the leading pair of the principal series and occur only in the spot spectrum.

Be Two lines at λ 3321 are present in the solar spectrum, but are very faint, having intensity −3.

Be^+ The ultimate lines, near λ 3131, have solar intensity 1, and are the only lines present.

B No atomic lines have been detected in the sun, although *B* is present in compounds.[1] Laboratory material is needed for this element.

C Lines of five multiplets are present. The strongest solar intensity is about 0; the lines are weakened or obliterated in the spot spectrum. This element also occurs in compounds.

N The presence of atomic *N* depends on two lines in the infra-red. The solar intensities are −2. *N* is also present in several compounds.

O Five lines belonging to the principal series are present. These range from intensity 2 to −1 in the disk, and are all weakened in the spot spectrum. *O* is also present in compounds.

Na Lines of the principal, diffuse, and sharp series are present; all are much strengthened in the spot spectrum.

Mg The one line arising from the lowest atomic level, which occurs in the visible region, is slightly strengthened in spots. Lines of the sharp and diffuse series show very little difference in intensity in the two spectra. The triplet whose leading line is at λ 5183 is outstanding among solar lines, being surpassed in intensity only by lines of Ca^+, *H*, and *Fe*. The λ 3800 group is also very strong in the solar spectrum.

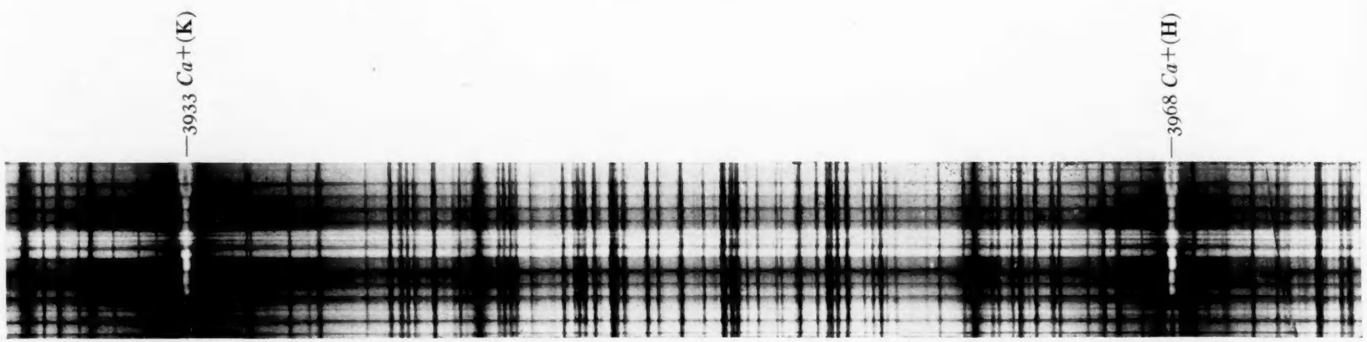
Mg^+ The pair at λ 4481 furnishes the only evidence for Mg^+. These lines are of intensity 0 in the solar spectrum and are both weakened in the spot spectrum, as announced by St. John in 1921.[2]

Al The ultimate lines at λ 3944 and λ 3961 are strong and become winged in the spot spectrum. The pair in the red, λ 6696 and λ 6698, and λ 5557 are also present; all are strengthened in the spot spectrum. No spot intensities are available for the group whose principal line is at λ 3092.

[1] Nicholson and Perrakis, *Mt. Wilson Contr.*, No. 370; *Astrophysical Journal*, **68**, 327, 1928.

[2] *Contributions from the Jefferson Physical Laboratory, Harvard University*, **15**, No. 29, 1921.

PLATE XIV



H and K Lines of Ionized Calcium in Solar and Spot Spectra

Al⁺ One line, λ 3900.680, may be present. It is faint, having a solar intensity −2.

Si Approximately thirteen multiplets are represented. The lines from lower atomic levels have the same intensities in disk and spot spectra. Those from higher levels are much weakened in the spot.

Si⁺ Two multiplets are represented in the solar spectrum. The two lines observable in the spot spectrum are obliterated.

S One multiplet may be present. This group is near λ 9200, and the strongest solar intensity is probably −1.

K Only the lines of the principal series are present in the solar spectrum. These are much strengthened in the spot spectrum.

Ca Conspicuous in the solar spectrum; all the lines are strengthened in the spot spectrum.

Ca⁺ H and K are the strongest lines in both solar and spot spectra. It is impossible to estimate the spot intensities of these lines, but no weakening can be detected. Plate XIV illustrates their appearance. H_3 and K_3 are bright in the spot region, which might make it difficult to notice any weakening. The strong group in the infra-red shows no diminution of intensity in the spot spectrum.

Sc Most of the lines are faint in the solar spectrum, the strong ultimate lines having solar intensity 2. Practically all are strengthened in the spot spectrum, those in the red arising from a low atomic level being conspicuous. Several spot lines are attributed to *Sc*.

Sc⁺ The violet lines are strong in the solar spectrum. Most of the lines differ very little in intensity in the spot spectrum, but a few from the higher atomic levels are weakened.

Ti The lines are all strengthened in the spot spectrum, those originating in low atomic levels being greatly increased in intensity. Many lines which appear only in the spot spectrum are due to *Ti*.

Ti⁺ Well represented in both disk and spot spectra, and most of the lines show but a slight difference in intensity in the two. A few arising from the higher atomic levels are weakened in the spot spectrum.

V Most of the lines are faint in the solar spectrum, but are greatly strengthened in the spot and therefore furnish identifications for a number of spot lines.

V⁺ Some strong multiplets are present in the violet region of the solar spectrum. Evidence based on the few available lines indicates no difference in intensity between disk and spot spectra.

Cr Many multiplets exist among the solar lines, the triplet near λ 4200 being conspicuous. The lines are strengthened in the spot spectrum, those of low atomic level showing the effect to a marked degree.

Cr⁺ The lines are numerous and are weakened in the spot spectrum.

Mn The ultimate lines are strengthened three or four units in the spot spectrum. Most of the lines are stronger in the spot spectrum than in that of the disk.

Mn⁺ Presence in the solar spectrum conclusively established by one multiplet in the ultra-violet. None of the lines occurs within the sun-spot region.

Fe This element gives the strongest solar lines, excepting those of Ca^+ and *H*. The lines arising from low atomic levels are strengthened in spots; those from higher levels show very little change from the solar intensity.

Fe⁺ The lines are all weakened or obliterated in the spot spectrum.

Co The ultimate lines lie in the ultra-violet and have solar intensities 4 or 5. Many of the lines in the spot region are faint. Those from low atomic levels show a slight increase of intensity in the spot; the others remain unchanged.

Co⁺ Two multiplets are present in the ultra-violet. The strongest line has solar intensity 0.

Ni The ultimate lines are strengthened in the spot spectrum. The effect decreases with increasing atomic level and lines from the highest level are slightly weakened.

Ni⁺ The multiplets lie mostly to the violet of the sun-spot region, but the stronger lines are present and produce solar lines of maximum intensity 2 or 3. The only unblended line for which a spot estimate could be made is marked "ob?" but the presence of a band line makes the estimate questionable.

Cu The ultimate lines have solar intensity 10. A few lines from higher atomic levels are present and have the same intensity in disk and spot.

Zn The lines in the spot region arise from high atomic levels and are much weakened in the spot spectrum. Additional multiplets are present in the ultra-violet.

Ga The two ultimate lines are present in the solar spectrum. One is blended with *Cr;* the other has solar intensity -1, but is masked in the spot spectrum by a neighboring winged line.

Ge This element is present, giving solar lines of intensity 0 or -1, on the average. The behavior of *Ge* in the spot spectrum is not known, since the only two lines within this region are disturbed by neighboring lines.

Rb Represented by two lines, the leading pair of the principal series, which occur only in the spot spectrum.

Sr The strongest lines are present, but faint, in the solar spectrum. They are strengthened in the spot spectrum. Several lines are found only in the spot spectrum.

Sr⁺ The ultimate lines λ 4077 and λ 4215, the first pair of the principal series, are conspicuous solar lines and are not weakened in the spot spectrum.

Sr^+ cont. Lines from higher atomic levels are also present and are weakened in the spot spectrum.

Y The strongest lines are very faint in the solar spectrum. They are considerably strengthened in the spot, and some of the fainter arc lines appear only in the spot spectrum.

Y^+ All the leading multiplets are present in the solar spectrum. Most of the lines show but little difference in intensity between disk and spot, although a few are slightly weaker in the spot spectrum.

Zr The strongest arc lines are of intensity -2 or -3 in the solar spectrum. These are strengthened in the spot spectrum, and several of the fainter arc lines appear only in the spot spectrum.

Zr^+ The strongest lines lie in the violet and have solar intensity 2. None of the lines shows much change in intensity when the disk and spot spectra are compared.

Cb Presence in the solar spectrum somewhat questionable, even the strongest lines being probably very faint. The multiplet evidence suggests that nine lines may be present. Unfortunately, the only lines within the spot region are seriously blended with those of other elements.

Cb^+ Six leading lines of a strong multiplet which have low excitation potentials are present in the ultra-violet region of the solar spectrum. The maximum solar intensity is about -1.

Mo Present in the solar spectrum, but faint, with intensities -2 or -3. The ultimate lines are badly blended. Other lines from low atomic levels are strengthened in the spot spectrum.

Mo^+ Two solar lines may be attributed to Mo^+ with some assurance. They are in the ultra-violet and have solar intensity -3.

Ru The ultimate lines are present in the solar spectrum, but are faint. Most of the lines are too weak to appear in the spot spectrum, but four lines from low atomic levels are slightly strengthened.

Rh The strongest lines are in the violet and coincide with solar lines of intensity -2 or -3. It is doubtful whether any but the strongest ultimate lines are present.

Pd Here again the strongest lines are to the violet of the spot region. The element is present with maximum solar intensity 0. Eighteen solar lines have been attributed to *Pd*, although some are blends.

Ag Only two lines are present, the first pair of the principal series. They lie in the violet, and the stronger has solar intensity 0.

Cd Only the resonance line λ 3261 is present in the solar spectrum.[1] Its intensity here is -1.

[1] St. John and Moore, *Publications of the Astronomical Society of the Pacific*, **39**, 314, 1927.

In The most persistent line, λ4511, is present in the spot spectrum only, as are the lines of *Li* and *Rb*. The second line of the pair is masked by $H\delta$, and no other line is present.

Sn Identification in the solar spectrum doubtful. Two solar lines of intensity −2 at λ 3330 and λ 3801 may be produced by this element. The strongest arc lines are masked in the sun and the absence of a strong line at λ 3009.138 causes some doubt as to the presence of *Sn*.

Sb Three strong arc lines in the violet appear to be present in the solar spectrum, but are very faint, having intensity −3.

Ba^+ The strongest lines are conspicuous in the solar spectrum and appear abnormally strong in the spot spectrum. (No lines of *Ba* are present in the spectrum either of the disk or of the spot.)

La^+ The strongest lines have solar intensity 1. Most of the lines are fainter and show no difference in intensity in the disk and spot spectra.

Ce^+ Numerous lines are present, with maximum intensity 0. There is no difference between the disk and spot intensities.

Pr^+ The presence of Pr^+ in the solar spectrum is now confirmed by its appearance in the chromosphere.[1] The maximum solar intensity is −1. Most of the lines are too faint to appear in the spot spectrum.

Nd^+ Practically all the strong lines are present in the solar spectrum, with maximum intensity 0 or −1. Many fainter ones can also be found. No difference between disk and spot intensity is indicated, but spot intensities have been estimated for very few of the lines.

Sa^+ The strongest lines have intensity about −1 in the solar spectrum and are nearly all present. Most of the lines are too faint to appear in the spot spectrum.

Eu^+ Lines of this element seem to be unusually strong in the solar spectrum and show no change of intensity in the spot. The strongest lines are of intensity 1. Six strong lines appear to be present.

Gd^+ Most of the stronger lines appear in the solar spectrum and have intensity about −1. The existence of this element in the flash spectrum lends additional confirmation to the identifications. A few lines present in the spot spectrum are unchanged in intensity.

Tb^+ Three faint solar lines, λ 3939.597, λ 3946.815, and λ 4278.553, may possibly be due to Tb^+, but the absence of other strong lines casts doubt upon the identifications. More evidence is needed to decide about its presence.

Dy^+ The strongest lines are present and have intensity −1 in the solar spectrum. Possibly some of the fainter lines may be identified in the sun when more is known about the spectrum. The few lines present in both disk and spot spectra are unchanged in intensity.

[1] G. Abetti, *Memorie della Società Astronomica Italiana*, **5**, 383, 1931.

Er^+ The two strongest lines, λ 3896.26 and λ 3906.34, are present in the solar spectrum, although one is blended with *Co*. The weaker line, solar intensity −1, is unblended.

Yb^+ Two strong lines, λ 3289 and λ 3694, due to the ionized atom, coincide with solar lines of intensities 4 and 3 and are probably present. These are much stronger than the other rare-earth lines, but the high velocity of outflow from spots indicates that a heavy element contributes to the production of λ 3694. The line at λ 3289 is due mostly to V^+.

Lu^+ The only rare-earth element except La^+, *Yb*, and Yb^+ for which identification in the sun rests on multiplet evidence. On this basis four strong Lu^+ lines are present[1] and two possibly present. The strongest solar intensity is probably −3. There is no spot material for these lines.

Hf^+ The persistent lines have maximum solar intensity −1 and are nearly all present. A few are masked. One line of solar intensity −2, for which there is a spot estimate, indicates no difference of intensity between the two spectra.

W The *raies ultimes* are present or accounted for and have solar intensity −1. The evidence afforded by two multiplets of low atomic level confirms the identifications. One unblended line is slightly strengthened in the spot spectrum.

Os At present one cannot say definitely whether or not any solar lines are due to *Os*. The evidence consists of only a few coincidences with faint solar lines.

Ir The laboratory data are so meager that it is impossible to decide whether the element is present or absent. Two strong arc lines, λ 3068 and λ 3220, may exist in the solar spectrum, but more evidence is needed to confirm the identifications.

Pt Analysis of the spectrum confirms its presence in the solar spectrum. Four strong lines from low atomic levels appear in the ultra-violet, the strongest having solar intensity 1.

Tl The evidence for the presence of *Tl* among the solar lines rests on one strong ultimate line at λ 3775 which has solar intensity −2. The multiplet includes two lines, the second of which is at λ 5350. It coincides with a faint solar line that is not strengthened in the spot spectrum and cannot therefore be attributed to *Tl*. This casts doubt on the presence of the element in the solar spectrum.

Pb The two strongest lines are present, but faint, in the solar spectrum, having intensities −2 and −3. One of these occurs in the spot region, but is blended in the spot spectrum.

[1] Moore, *Publications of the Astronomical Society of the Pacific*, **42**, 346, 1930.

A detailed discussion of the elements present in the solar and spot spectra and an explanation of many of the absences has already been published.[1] The elements not appearing in the sun[2] divide themselves into two groups: first, *P*, *Se*, *Ma*, *Te*, *Il*, *Tu*, *Ta*, *Po*, *Rn*, *Ac*, *Th*, *Pa*, and *U*, whose spectra have not been sufficiently observed over a wide spectral range to permit any decision as to their presence or absence; second, *F*, *Ne*, *Cl*, *Ar*, *As*, *Br*, *Kr*, *I*, *Xe*, *Cs*, *Ho*, *Re*, *Au*, *Hg*, *Bi*, and *Ra*, whose spectra have been investigated and whose lines all appear to be absent from the solar spectrum.

The rare earths, however, require comment, because Mr. King has kindly furnished much additional material in advance of publication as a further aid in their identification. A study of these elements was begun in 1928,[3] but at that time no lines of neutral atoms were found in the solar spectrum. Since then Mr. King has made temperature classifications for *Hf*[4], *Ce*, and *Pr*[5] and for *Eu*, *Gd*, *Tb*, *Dy*, and *Ho*.[6] He has also selected the strongest arc and spark lines of *Nd* and *Sa*.[7] These new data confirm the former conclusion that the rare earths are present in both sun and spot, but only in the ionized state, with the exception of *Yb*.[8] A few coincidences of arc lines of *Dy*, *Tb*, and *Ho* with solar lines are regarded as accidental because of the absence of other strong lines. For *Er* the arc spectrum is so incompletely observed that nothing can be said about its be-

[1] St. John, Moore, Ware, Adams, and Babcock, *loc. cit.*; Russell, *Mt. Wilson Contr.*, No. 383; *Astrophysical Journal*, **70**, 11, 1929; St. John, *Mt. Wilson Contr.*, No. 385; *Astrophysical Journal*, **70**, 160, 1929.

[2] Russell, *Mt. Wilson Contr.*, No. 383; *Astrophysical Journal*, **70**, 11, 1929; St. John, *Mt. Wilson Contr.*, No. 385, *Astrophysical Journal*, **70**, 160, 1929.

[3] St. John and Moore, *Mt. Wilson Contr.*, No. 364; *Astrophysical Journal*, **68**, 93, 1928.

[4] *Mt. Wilson Contr.*, No. 384; *Astrophysical Journal*, **70**, 105, 1929.

[5] *Mt. Wilson Contr.*, No. 368; *Astrophysical Journal*, **68**, 194, 1928.

[6] *Mt. Wilson Contr.*, No. 414; *Astrophysical Journal*, **72**, 221, 1930.

[7] Unpublished material, 1930.

[8] From recent data by Meggers (unpublished) and by King (*Mt. Wilson Contr.*, No. 439; *Astrophysical Journal*, **74**, 328, 1931) there is evidence that the two strongest ultimate lines of *Yb*, λ 3987 and λ 5556, are present in the solar spectrum and are strengthened in the spot spectrum. This is the only known case of the presence in the sun of a rare-earth element in the neutral state. The ionization potential of *Yb* is 6.23 volts, according to unpublished material by Meggers and Russell (1932).

havior in the sun.[1] The solar identifications of the rare earths are also confirmed by the appearance of the lines in the flash spectrum, where they can be detected by their peculiar characteristic behavior, as described by D. H. Menzel.[2] They do not reverse into absorption lines suddenly at the limb, but show in emission far in on the solar disk. Mr. Menzel has kindly furnished an unpublished list of flash lines showing this characteristic appearance, which has been of great assistance in identifying these elements.

The multiplet material on which the foregoing discussion is based is contained in a complete table of solar multiplets, arranged by elements in the order of increasing atomic number, now in preparation. The table will include all the solar and spot lines of known multiplet designation between λ 2975 and λ 7000. A table of term values and corresponding excitation potentials for the various elements will be appended. This multiplet table is too extensive to include here, but it is hoped that it will soon be available for distribution.

Table XVIII gives references to spectroscopic material used for this work. The elements, neutral and ionized, are arranged in the order of atomic number. The tabular numbers refer to the bibliography in this article. For the most part only the more recent references are given. Those already listed in the *Revised Rowland*[3] and in the bibliography by Russell[4] are not repeated here.

4. APPLICATION OF THE IONIZATION THEORY

The importance of the Saha ionization theory is well known to those engaged in the interpretation of solar and stellar spectra. The fundamental equation

$$\log \frac{x}{1-x} = -\frac{5040I}{T} + \frac{5}{2}\log T - 6.5 - \log P_e \tag{1}$$

shows that a decrease in the electron pressure P_e increases the degree of ionization $x/(1-x)$ in the same proportion for all elements, where

[1] St. John and Moore, *op. cit.*

[2] *Publications of the Astronomical Society of the Pacific*, **39**, 359, 1927.

[3] St. John, Moore, Ware, Adams, and Babcock, *loc. cit.*

[4] *Mt. Wilson Contr.*, No. 383; *Astrophysical Journal*, **70**, 11, 1929.

$x/(1-x)$ is the ratio of the fraction of ionized atoms to that of neutral atoms. A decrease in temperature T decreases the degree of ionization by an amount which varies with the ionization potential I.

In some of the early work on the sun-spot spectrum by Hale, Adams, and Gale[1] it was suggested that a temperature difference would explain the behavior of the lines.

Applying the ionization theory to the sun-spot spectrum, M. N. Saha[2] made predictions about the behavior of the alkali metals which were completely verified. The theory was extended to mixtures of elements by Russell,[3] who proved that ". . . . the degree of ionization is higher for an element of easy than for one of difficult ionization; the relation between the two depends only on $I_2 - I_1$ and on T, and not on the pressure or on the relative concentrations." Russell also showed that enhanced lines of easily ionized elements may be produced with less excitation than is required for the arc lines of elements difficult to ionize.

The effect of decreased pressure on enhanced lines, particularly the Mg^+ lines at λ 4481, has been discussed by St. John in the application of the theory of ionization to solar faculae.[4]

It is customary to assume that the complex atmosphere of a star may be replaced by a layer of definite thickness and of uniform temperature and pressure, transparent except for line absorption and overlying a definite photospheric surface which radiates like a black body of effective temperature T_e. E. A. Milne has shown that this assumption gives surprisingly accurate results. The quantity of material in the fictitious homogeneous atmosphere is equal to that in the actual atmosphere down to the level at which the integrated opacity τ is about one-third, and the mean effective electron pressure P_e is substantially one-half, of that at the bottom. A. Pannekoek[5] has shown that, on account of excitation by radiation, the photo-

[1] *Mt. Wilson Contr.*, No. 11; *Astrophysical Journal*, **24**, 185, 1906.

[2] *Philosophical Magazine*, **40**, 809, 1920; **41**, 267, 1921; *Proceedings of the Royal Society*, A, **99**, 135, 1921.

[3] *Mt. Wilson Contr.*, No. 225; *Astrophysical Journal*, **55**, 119, 1922.

[4] *Contributions from the Jefferson Physical Laboratory, Harvard University*, **15**, No. 29, 1921.

[5] *Bulletin of the Astronomical Institutes of the Netherlands*, **3**, 207 (No. 110), 1926.

spheric temperature primarily determines the degree of ionization throughout the atmosphere. If T is the effective temperature of the photosphere, the ionization in a region where the temperature is T_1 and the electron pressure P_e is given by

$$\log \frac{x}{1-x} = -\frac{5040I}{T} + \frac{5}{2}\log T - 6.5 - \log P_e + \frac{3}{2}\log \frac{T_1}{T} + \log \frac{1}{2} . \quad (2)$$

In Pannekoek's formula the value of P_e, following Milne, should be taken as one-half that at the base of the atmosphere. Saha's formula, with the temperature and pressure prevailing at the assumed surface of the photosphere, gives nearly the same results. This correction for photospheric radiation takes into account the most important deviation from a state of thermodynamic equilibrium that has yet been theoretically discussed.

R. H. Fowler[1] has pointed out that the second member of (2) must be increased by $\log [\sigma B'(T)/B(T)]$, where σ is the symmetry number of the electron and is equal to 2; $B'(T)$ and $B(T)$ are the partition functions of the ionized and neutral atoms, respectively, defined by

$$B(T) = g_0 + g_1 e^{-\frac{\epsilon_1}{kT}} + g_2 e^{-\frac{\epsilon_2}{kT}} , \quad (3)$$

where g_1, g_2, etc., are the statistical weights of the different states and ϵ_1, ϵ_2, etc., their energies of excitation. The value of g is the product of the multiplicity of the term, 1, 2, 3, for singlets, doublets, triplets, etc., by 1, 3, 5, 7, for S-, P-, D-, F-terms, respectively. For example, $g=9$ for a ^{3}P-term. The value must be doubled for all terms arising from electron configurations involving two equivalent *s*-electrons.

In the reversing layer T_1/T is about 0.88. Log σ and log $\frac{1}{2}$ cancel each other, and equation (2) may be written

$$\log \frac{x}{1-x} = -\frac{5040I}{T} + \frac{5}{2}\log T - 6.59 - \log P_e + \log \frac{B'(T)}{B(T)} . \quad (4)$$

Strong evidence in support of the ionization theory is found in the behavior of the alkali metals, where the correlation between strength-

[1] *Statistical Mechanics* (1929), p. 334.

ening in the spot spectrum and the ionization potentials is conspicuous. Thus lines of three series of *Na* (I.P. 5.11 volts) appear in the spot spectrum; for *K* (I.P. 4.33) only those of the principal series are present; *Rb* (I.P. 4.13) is represented entirely by the principal pair of the principal series, which appears only in the spot spectrum; while *Cs* (I.P. 3.86) is absent, being completely ionized even in the spot spectrum. *Li* (I.P. 5.36), like *Rb*, appears only in spots, being far less abundant than *Na* or *K*.[1]

The behavior of the alkaline earths confirms the theory still further. All the lines are strengthened in the spot, and the relative intensities in the disk and spot are again a function of the ionization potential. For *Ca* (I.P. 6.09) the lines of the principal series show the greatest increase of intensity in the spot. The same is true to a less degree of *Sr* (I.P. 5.65), for which most of the lines are very faint in the disk spectrum and some of the fainter ones appear only in the spot spectrum. *Ba* (I.P. 5.19) is so highly ionized that its lines are absent even from the spot spectrum—a fact not yet fully explained. The enhanced lines of these elements show no decrease of intensity in the spot. Three favorable conditions combine to produce H and K of Ca^+, the most outstanding lines of the whole spectrum. They are the lines "which the atoms of one of the most abundant constituents of the solar atmosphere absorb most powerfully when in that state in which, under solar conditions, they are most likely to occur."[2] Most of the atoms of *Ca* and *Sr* and all of *Ba* must be ionized above the spot.

The ultimate line, λ 4571, of *Mg* (I.P. 7.61) is slightly strengthened in the spot spectrum, while the arc lines of *Zn* (I.P. 9.36) are weakened in the spectrum of the spot.

For the elements from *Sc* to *Ni* inclusive, the behavior in the spot spectrum is directly comparable with that in laboratory spectra produced at various temperatures, as in the furnace and arc. Lines of *Sc* (I.P. 6.57) are faint in the solar spectrum but strengthen in the spot spectrum, the flame lines showing the greatest increase. The enhanced lines, which are strong in the arc, show very little change of intensity in the spot. *Ti* (I.P. 6.80) and *V* (I.P. 6.76), owing to

[1] Russell, *Mt. Wilson Contr.*, No. 383; *Astrophysical Journal*, **70**, 11, 1929.

[2] Russell, *Mt. Wilson Contr.*, No. 225; *Astrophysical Journal*, **55**, 119, 1922.

their similarity, may be treated together. Nearly all the arc lines appear in the furnace, and the furnace lines show the greatest increase in spot intensity. The enhanced lines which are difficult to produce in the furnace show a tendency to weaken in the spot. The behavior of *Cr* (I.P. 6.74), *Mn* (I.P. 7.40), and *Fe* (I.P. 7.83) is similar. The flame lines are strengthened in the spot, the arc lines show but little change, and the enhanced lines are weakened. For these elements the enhanced lines are absent from the furnace spectrum and are faint in the arc. For *Co* (I.P. 7.81) and *Ni* (I.P. 7.64) the low-temperature lines are strengthened in the spot. The *Ni* lines appearing only in the arc, King's class V lines, are slightly weakened. *Y* (I.P. 6.5) and *Zr* (I.P. 6.92) are faint in the solar spectrum, but the low-temperature lines strengthen in the spot spectrum.

In the rare earths, agreement with the ionization theory is again found. A comparison of the spectra of La^+ and Lu^+ shows that the ultimate enhanced lines of the rare earths move very slowly toward the violet as the ionization potentials increase. The strong enhanced lines which are easily produced in the furnace arise from low atomic levels and are comparable with the ultimate lines of Ti^+ near λ 3700, which are strong in furnace spectra. In the rare earths the lines from low atomic levels cover a large range of spectrum. That they are produced by ionized atoms is proved conclusively by the fact that they disappear when a mixture with *Cs* is used. Ionization potentials determined experimentally for *La*, *Ce*, *Pr*, *Nd*, *Sa*, *Gd*, *Tb*, *Dy*, and *Yb*[1] range from 5.5 volts for *La* to 7.1 for *Yb*.[2] The consistency in the behavior of the rare earths in the furnace and in solar and spot spectra, where they show no change of intensity, and the intensity of the lines in the low-lying chromosphere also indicate low ionization potentials, perhaps even lower for some of the elements (*Ce*?) than the published values. These results are all in harmony with the fact that the rare earths appear in the sun only in the ionized state.[2]

In addition to these results bearing on the ionization theory, a few general facts deduced from a study of the solar lines may be reviewed, which explain the behavior of several elements not previously mentioned.

[1] Rolla and Piccardi, *Philosophical Magazine*, **7**, 286, 1929.

[2] See p. 306, n. 8.

The importance of excitation potentials in explaining solar phenomena has been discussed in papers by Russell[1] and by St. John.[2] A high excitation potential is unfavorable to the appearance of a solar line. The accessible lines of *B*, *F*, *Ne*, P^+, *Cl*, *A*, *Se*, *Br*, *Kr*, *Te*, *I*, *Xe*, Cs^+, *Au*, *Hg*, and Bi^+, all of which are absent from the solar spectrum, have excitation potentials greater than 4.5 volts. For *He* the value increases to 20 volts.

The presence of lines of high excitation potential, when they occur in the solar spectrum, is due to the abundance in the solar atmosphere of the elements that produce them.[3] *H* is very exceptional and must be exceedingly abundant; the lines are among the strongest and have an excitation potential of 10 volts. They are much weakened in the spot spectrum, however, which may be partly due to the formation of *H* compounds in the spot. The presence of lines of *C*, *N*, *O*, and *S*, all having high excitation potentials, is also explained on the basis of abundance. The lines of these elements are all faint in the solar spectrum. Those of *C* and *O* are weakened in the spot spectrum. The *N* and *S* lines are in the infra-red.

With very few exceptions, elements having enhanced lines of low excitation potential in the accessible regions show them in the solar spectrum.[4] For many such elements the persistent lines lie too far to the violet, and only lines from higher atomic levels are available. For example, the ionization potential of *Cs* is so low that the atoms are all ionized and no lines of the neutral atom occur even in the spot spectrum. The only lines of Cs^+ in the accessible region of the solar spectrum have an excitation potential of 15 volts; therefore the very faint solar lines attributed to Cs^+ in the *Revised Rowland* must be erroneously identified.

Lastly, it has been shown that elements of even atomic number are on the average about ten times as abundant as those of odd atomic number.[5] It is noticeable among the rare earths that Ce^+

[1] *Mt. Wilson Contr.*, No. 383; *Astrophysical Journal*, **70**, 11, 1929.

[2] *Mt. Wilson Contr.*, No. 385; *Astrophysical Journal*, **70**, 160, 1929; *Mt. Wilson Contr.*, Nos. 389 and 390; *Astrophysical Journal*, **70**, 312, 319, 1929.

[3] Russell, *Mt. Wilson Contr.*, No. 383; *Astrophysical Journal*, **70**, 11, 1929; St. John, *Mt. Wilson Contr.*, No. 385; *Astrophysical Journal*, **70**, 160, 1929.

[4] Russell, *Mt. Wilson Contr.*, No. 383; *Astrophysical Journal*, **70**, 11, 1929.

[5] *Ibid.*

(At. No. 58), Nd^{+} (At. No. 60), and Sa^{+} (At. No. 62) are represented by a large number of lines.

When more has been done in the laboratory, many questions of identification which at present are doubtful can be conclusively settled. Also many more faint solar lines can probably be attributed to the rare earths.

5. DETERMINATION OF EFFECTIVE TEMPERATURE OF SUN-SPOTS AND THE PRESSURE AND AMOUNT OF MATERIAL ABOVE THEM

Approximate formulae giving the relative intensities of the lines of a multiplet have been derived by several investigators.[1] These formulae, which are based on the correspondence principle and depend only on the quantum numbers of the lines, give the relative numbers of fictitious resonators involved in the production of the spectral lines. Assuming that the numbers of these resonators are proportional to the numbers of atoms acting, Adams, Russell, and the writer calibrated the Rowland intensity scale[2] with the aid of the multiplet material collected during the revision of the *Rowland Table*. In all, 228 multiplets, 1288 lines, and 312 increments—ΔR and $\Delta \log N$—were used, ΔR being the difference between two of Rowland's estimated intensities, and $\Delta \log N$ the corresponding change in the theoretical numbers of atoms producing the two lines involved. A curve of $\Delta \log N/\Delta R$ as a function of R was then drawn, from which $\log N$ was found. A comparison of the theoretical values of $\Delta \log N$ with those from the curve corresponding to the observed values of ΔR indicated a change in the Rowland scale with wave-length. On the assumption that

$$\log N = B \log A \ , \tag{5}$$

where A depends on the Rowland intensity, R, and B on the wave-length, empirical tables were made of the value of $\log A$ corresponding to each Rowland intensity and of B for the whole range of

[1] R. de L. Kronig, *Zeitschrift für Physik*, **31**, 885, 1925; Sommerfeld and Hönl, *Sitzungsberichte der Preussichen Akademie der Wissenschaften*, p. 141, 1925; Russell, *Proceedings of the National Academy of Sciences*, **11**, 314, 1925.

[2] *Mt. Wilson Contr.*, No. 358; *Astrophysical Journal*, **68**, 1, 1928.

wave-lengths. The probable error of log N derived from a single Rowland line is ± 0.32.

With the aid of this calibration, Adams and Russell have estimated the relative numbers of atoms in stellar atmospheres which are active in producing a given line.[1] If the intensities of stellar lines are recorded on the Rowland scale, temperatures can also be determined[2] by assuming that thermodynamic equilibrium exists and that atoms at different levels are equally effective in producing a line. The equation for the comparison of the same lines in different stars is

$$\log \frac{N'_x}{N_x} = \log \frac{N'_0}{N_0} + 5040\, E_x\left(\frac{1}{T} - \frac{1}{T'}\right) + r \log \frac{D'}{D}\,, \qquad (6)$$

where N'_x/N_x is the ratio of the numbers of atoms producing the line in the two stars obtained from the calibration formula, and N'_0/N_0 is the ratio of the numbers of neutral atoms of the element considered above equal areas of the photospheres of the respective stars. T and T' are the temperatures of the two stars, r the degree of ionization, and D'/D a function of the effective electron pressures for a given element in the two stars.

If the foregoing assumptions hold for any one element and any star, log (N'_x/N_x), which for brevity may be called Y, should be a linear function of the excitation potential E_x. Hence, for any star the values of Y for a given element plotted against E_x should define a straight line having the slope

$$S = \frac{\Delta Y}{\Delta E_x} = 5040\left(\frac{1}{T} - \frac{1}{T'}\right)\,. \qquad (7)$$

Here T is the temperature of the sun's atmosphere, and T' that of the star's atmosphere. The slope S should be the same for all elements in any star; but errors of observation enter and a weighted mean should be taken. If there are n values of Y and e is the extreme range of E_x, the weight is approximately ne^2.

The estimated intensities of some six thousand lines in the spot spectrum may be used by the method just outlined to determine the temperature of sun-spots. Since these estimates are on the Rowland

[1] *Mt. Wilson Contr.*, No. 359; *Astrophysical Journal*, **68**, 9, 1928. [2] *Ibid.*

scale, no correction for scale is required. One serious difficulty arises, however, when the method is applied to the sun-spot spectrum. It is a known but unexplained fact that for elements such as *Ti* and *V*,

TABLE VIII

ELEMENTS USED IN DETERMINING SLOPE

$$Y=\log \frac{N'_x}{N_x}$$

Element	E_x	No.	Y
Ca.....	0.00	2	+1.16
	1.88	11	0.80
	2.51	35	0.75
	2.80	13	+0.62
		61	±0.30
Ti......	0.02	33	+1.42
	0.82	42	1.24
	1.05	21	1.34
	1.44	23	1.28
	1.81	28	1.17
	2.10	13	1.15
	2.37	22	0.94
	3.35	10	+0.94
		192	±0.42
Cr......	0.00	3	+0.53
	0.98	44	0.93
	2.59	21	0.52
	2.94	23	0.59
	3.12	11	0.67
	3.40	15	+0.60
		117	±0.22
Mn.....	0.00	5	+1.21
	2.18	12	0.51
	2.95	35	0.40
	3.23	9	0.43
	3.81	6	+0.60
		67	±0.24
Fe......	0.06	25	+0.81
	0.96	38	.67
	1.54	38	.40
	2.21	42	.26
	2.43	16	.21
	2.58	15	.29
	2.72	15	.04
	2.84	43	.20
	2.97	15	.25
	3.24	26	.15
	3.38	34	.14
	3.63	15	.04
	3.92	24	.02
	4.20	32	.05
	4.51	11	.21
	4.78	6	+0.14
		395	±0.21
Ni.....	1.67	12	+0.37
	1.91	15	+ .42
	3.35	13	.00
	3.56	15	− .18
	3.66	29	− .25
	3.74	10	− .22
	3.85	22	− .05
	4.08	13	− .15
	4.14	13	− .36
	4.27	11	−0.25
		153	±0.20

whose lines are strengthened in the sun-spot spectrum, the fainter lines increase in intensity more than the stronger lines. The use of an element like *V* would therefore introduce a large error because practically all the lines are faint in the solar spectrum. For *Ti* the

error is less serious because both strong and weak lines occur in the spectrum of the disk.

The material selected for the determination of the spot temperature consists of the unblended lines of *Ca*, *Ti*, *Cr*, *Mn*, *Fe*, and *Ni*. The data are summarized in Table VIII, which gives for each element the mean excitation potential, the number of lines included in the mean, and the mean value of Y obtained from the calibration tables. The last line under each element gives the total number of lines and the average deviation of the individual values of Y from the mean for each group. Similar data for elements not used in finding the slope appear in Table IX.

Plots of Y against E_x indicate at once the elements best suited to the purpose. *Fe*, with 395 lines covering a large range of excitation potential and intensity, has the best-determined slope. The values of Y for *Ti* depend on 192 estimates and give a good slope. For *Ca*, *Cr*, and *Mn* the slopes are based on fewer lines, but the points are sufficiently well determined to receive some weight. *Ni* is a little discordant; but the available range in excitation potential is limited, since the *Ni* lines from low atomic levels are to the violet of the sunspot region. The results for the slopes and other relevant data are in Table X. With Milne's value of 5740° K for the photospheric temperature and the weighted mean $S = -0.190 \pm 0.010$, equation (7) gives 4720° ± 40° K for the mean effective temperature of the spot. The probable errors are a measure of the accordance of the observations and not of the absolute precision of the results. The calculated temperature agrees well, however, with that determined by Pettit and Nicholson[1] from the spectral energy-curve of sunspots. For an assumed solar temperature of 5955° K, their value, corrected for scattered light, is 4750° K.

Adams and Russell[2] have also determined the electron pressures and the abundance of material in several stars relative to the sun, as follows: From equations (7) and (6),

$$Y = Y_0 + SE_x \,, \tag{8}$$

[1] *Mt. Wilson Contr.*, No. 397; *Astrophysical Journal*, **71**, 153, 1930.

[2] *Mt. Wilson Contr.*, No. 359; *Astrophysical Journal*, **68**, 9, 1928.

TABLE IX

$$Y = \log \frac{N'_x}{N_x}$$

Element	E_x	No.	Y
Na.....	2.09	7	+1.00
		7	±0.34
Mg.....	0.00	1	+0.51
	2.70	3	+0.03
	4.33	8	−0.07
		12	±0.08
Mg⁺....	8.83	2	−1.57
		2	±0.35
Si......	1.90	2	0.00
	4.93	10	−0.92
		12	±0.19
Si⁺.....	8.08	2	−2.20
		2	±0.22
K......	0.00	2	+1.00
		2	±0.18
Sc......	0.09	12	+1.79
	1.43	13	1.54
	1.88	7	+1.20
		32	±0.61
Sc⁺.....	0.31	3	0.00
	0.60	10	−0.06
	1.47	16	−0.31
		29	±0.17
Ti⁺.....	1.14	43	−0.32
	1.57	9	0.28
	2.18	8	0.32
	3.10	4	−0.40
		64	±0.28
V......	0.03	20	+1.70
	0.28	37	1.83
	1.05	31	1.61
	1.20	4	2.03
	2.73	2	+0.58
		94	±0.53
V⁺.....	1.64	13	0.00
	2.03	4	−0.17
		17	±0.06
Cr⁺.....	3.09	8	−0.16
	3.83	15	0.83
	4.07	15	−0.76
		38	±0.32
Fe⁺.....	2.63	8	−0.43
	2.84	31	0.92
	3.48	17	−1.02
		56	±0.32
Co......	0.59	18	+0.50
	1.62	18	+0.29
	2.68	7	0.00
	3.43	11	0.00
	4.10	5	−0.12
		59	±0.24
Cu.....	1.55	3	+0.22
	3.78	2	0.00
		5	±0.11
Zn.....	4.02	3	−1.05
	5.77	1	−0.44
		4	±0.02
Sr......	0.00	1	+1.05
	1.78	3	+0.86
		4	±0.22

TABLE IX—*Continued*

Element	E_x	No.	Y
Sr^+	0.00	2	+0.16
	2.93	1	−0.58
		3	±0.11
Y	0.03	6	+1.27
	1.37	5	+0.37
		11	±0.41
Y^+	0.27	8	−0.08
	1.02	9	0.32
	1.78	2	−0.29
		19	±0.26
Zr	0.07	7	+1.82
	0.63	11	1.51
	1.47	2	+0.63
		20	±0.51

Element	E_x	No.	Y
Zr^+	0.75	8	−0.07
	1.59	5	−0.24
	2.42	1	0.00
		14	±0.17
Mo	1.33	3	+0.91
		3	±0.23
Ru	0.87	3	+1.00
	1.20	1	+0.63
		4	±0.38
Ba^+	0.40	3	+0.33
	2.50	1	0.00
		4	±0.05
La^+	0.13	8	0.00
	0.35	3	0.00
		11	0.00

TABLE X

VALUES OF SLOPE USED IN CALCULATING SUN-SPOT TEMPERATURE

Element	No. Lines	ne^2	Wt.	Slope
Ca	61	477	5	−0.200
Ti	192	2125	21	.167
Cr	117	1352	14	.147
Mn	67	970	10	.192
Fe	395	8780	88	.188
Ni	153	1032	10	−0.305
Weighted mean	....	....	....	−0.190±0.010

where $Y_0 = \log (N_0'/N_0)$. Similarly, for ionized atoms

$$Y = Y_1 + SE_x\,, \tag{9}$$

where $Y_1 = \log (N_1'/N_1)$. Here N_0'/N_0 is the relative abundance of neutral atoms in their normal state in the spot compared with the disk, and N_1'/N_1 the corresponding ratio for ionized atoms.

Pannekoek's equation gives for the disk

$$\log \frac{N_1}{N_0} = \log \frac{x}{1-x} = -\frac{5040 I}{T} + \tfrac{5}{2} \log T - 6.59 - \log P_e + \log \frac{B'(T)}{B\,(T)} \tag{10}$$

and for the spot

$$\log \frac{N_1'}{N_0'} = \log \frac{x'}{1-x'} = -\frac{5040 I}{T'} + \tfrac{5}{2} \log T' - 6.59 - \log P_e' + \log \frac{B'(T')}{B\,(T')}\,. \tag{11}$$

By subtracting and comparing neutral and ionized atoms of the same element,

$$\log \frac{P_e'}{P_e} = Y_0 - Y_1 + \tfrac{5}{2} \log \frac{T'}{T} + 5040 I \left(\frac{1}{T} - \frac{1}{T'}\right) + \log \frac{B'(T')}{B\,(T')} - \log \frac{B'(T)}{B\,(T)}\,. \tag{12}$$

With $T = 5740^\circ$ and $T' = 4720^\circ$, this equation becomes

$$\log \frac{P_e'}{P_e} = Y_0 - Y_1 - 0.190 I - 0.20 + \log \frac{B'(T')}{B\,(T')} - \log \frac{B'(T)}{B\,(T)}\,. \tag{13}$$

For each of eight elements a sufficient number of lines from both the neutral and ionized atoms appear in the spot spectrum to make the evaluation of Y_0 and Y_1 possible. The elements, ionization potentials, weights, values of Y_0, Y_1, and $\log\,(P_e'/P_e)$ are in Table XI.

TABLE XI

El.	I	Wt.	Y_0	Y_1	$\log \frac{P_e'}{P_e}$
Sc	6.57	1	+1.76	+0.01	+0.28
Ti	6.80	2	1.47	− .04	+ .03
V	6.76	1	1.82	+ .29	+ .03
Cr	6.74	2	1.12	+ .05	− .36
Fe	7.83	2	0.77	− .31	− .61
Sr	5.65	1	1.16	+ .10	− .19
Y	6.5	1	0.98	− .07	− .42
Zr	6.92	1	+1.63	+0.09	−0.04

The material for *Ti*, *Cr*, and *Fe* is better than that for the other elements and has therefore been assigned greater weight. The adopted results are

$$\log \frac{P_e'}{P_e} = -0.22 \pm 0.07\,; \qquad \frac{P_e'}{P_e} = 0.60 \pm 0.10\,.$$

The electron pressure over the spot is therefore *lower* than that over the disk.

The "level of ionization," I_0, has been defined by Russell[1] as a quantity such that atoms of ionization potential I_0 and for which $B'(T)=B(T)$ are just 50 per cent ionized. Under these conditions $x/(1-x)=1$, $\log [x/(1-x)]=0$, and Pannekoek's equation gives for the disk and spot, respectively,

$$\frac{5040 I_0}{T}=\tfrac{5}{2}\log T-6.59-\log P_e\,, \tag{14}$$

$$\frac{5040 I_0'}{T'}=\tfrac{5}{2}\log T'-6.59-\log P_e'\,, \tag{15}$$

where I_0' is the level of ionization for the spot. By subtraction,

$$5040\left(\frac{I_0'}{T'}-\frac{I_0}{T}\right)=\tfrac{5}{2}\log\frac{T'}{T}-\log\frac{P_e'}{P_e}\,. \tag{16}$$

For the disk, $I_0=8.5\pm0.1$.[2] By substituting $T=5740°$, $T'=4720°$, and $\log (P_e'/P_e)=-0.22$, I_0', the level of ionization for the spot, comes out as 7.0 ± 0.1 volts.

With these values of I_0 and I_0' and the temperatures, it is easy to calculate $\log x$, $\log x'$, $\log (1-x)$, and $\log (1-x')$ for all elements whose ionization potentials are known. For any element, equation (4) may be written for the disk

$$\log\frac{x}{1-x}=-\frac{5040}{T}(I-I_0)+\log\frac{B'(T)}{B(T)}=0.878(8.5-I)+\log\frac{B'(T)}{B(T)}\,. \tag{17}$$

Similarly, for the spot

$$\log\frac{x'}{1-x'}=-\frac{5040}{T'}(I-I_0')+\log\frac{B'(T')}{B(T')}=1.068(7.0-I)+\log\frac{B'(T')}{B(T')}\,. \tag{18}$$

There is strong evidence of like distribution of material over the disk and spot.[3] Hence it may safely be assumed that the *relative*

[1] *Mt. Wilson Contr.*, No. 383; *Astrophysical Journal*, **70**, 11, 1929.

[2] Russell, *Mt. Wilson Contr.*, No. 383, p. 44; *Astrophysical Journal*, **70**, 54, 1929, and unpublished material.

[3] St. John, *Mt. Wilson Contr.*, Nos. 48, 54, 69, and 278; *Astrophysical Journal*, **32**, 36, 1910; **34**, 57 and 131, 1911; **37**, 322, 1913; **60**, 32, 1924.

abundance of material is the same over both. If N is the total number of atoms of a given element per square centimeter over the disk, and N' is the same quantity for the spot,

$$\frac{N'}{N}=C\ , \tag{19}$$

a constant, which is the same for all elements. Further,

$$N_0=N(1-x)\ , \qquad N_1=Nx\ , \tag{20}$$

where $1-x$ and x are the fractions of neutral and ionized atoms, respectively, over the disk; and similarly for the spot,

$$N_0'=N'(1-x')\ , \qquad N_1'=N'x'. \tag{21}$$

Then

$$Y_0=\log\frac{N_0'}{N_0}=\log C+\log\frac{1-x'}{1-x} \tag{22}$$

and

$$Y_1=\log\frac{N_1'}{N_1}=\log C+\log\frac{x'}{x}\ . \tag{23}$$

The values of Y_0 and Y_1 may be calculated from (8) and (9); $\log\,[(1-x')/(1-x)]$ and $\log\,(x'/x)$, from (17) and (18). Hence $\log C$ may be determined from (22) and (23).

Table XII gives the detailed data used to find $\log C$. The weights represent general estimates of the reliability of the data. The final value of $\log C$ for neutral atoms of nineteen elements is $+0.22\pm0.03$; for ionized atoms of eleven elements, $+0.26\pm0.04$. The adopted value from both neutral and ionized atoms is

$$\log C=+0.23\pm0.03\ ;$$

hence $N'/N=1.70\pm0.11$, which indicates a greater amount of material over the spot than over the disk.

Additional data similar to those in Table XII are given in Table XIII. These were not used to find $\log C$, but for a purpose described later.

The relative intensities of lines in the spot spectrum compared with those of the disk may be interpreted in terms of the three quantities, temperature, pressure, and abundance of material. The three

TABLE XII

El.	I.P.	Disk		Spot		Ionized Atom, Eq. (23)				Neutral Atom, Eq. (22)			
		$\log x$	$\log (1-x)$	$\log x'$	$\log (1-x')$	$\log \frac{x'}{x}$	Y_1	$\log C$	Wt.	$\log \frac{1-x'}{1-x}$	Y_0	$\log C$	Wt.
Na	5.11	0.00	−2.66	−0.01	−1.73	−0.01	......	......	.....	+0.93	+1.40	+0.47	1
Mg	7.61	− .07	0.85	0.74	0.09	0.67	+0.11	+0.78	½	0.76	+0.68	− .08	1
Si	8.14	.25	0.35	1.44	0.01	1.19	− .66	.53	½	0.34	+0.08	− .26	1
K	4.33	.00	3.36	0.00	2.55	0.00	......	......	.....	0.81	+1.00	+ .19	½
Ca	6.09	.00	2.12	0.04	1.03	0.04	......	......	.....	1.09	+1.20	+ .11	2
Sc	6.57	.01	1.75	0.12	0.61	0.11	+ .01	.12	2	1.14	+1.76	+ .62	1
Ti	6.80	.01	1.53	0.19	0.44	0.18	− .04	.14	4	1.09	+1.47	+ .38	4
V	6.76	.02	1.35	0.28	0.32	0.26	+ .29	.55	1	1.03	+1.82	+ .79	1
Cr	6.74	.02	1.27	0.29	0.32	0.27	+ .05	.32	2	0.95	+1.12	+ .17	3
Mn	7.40	.07	0.85	0.73	0.09	0.66	......	......	.....	0.76	+1.01	+ .25	2
Fe	7.83	.11	0.64	1.00	0.05	0.89	− .31	.58	2	0.59	+0.77	+ .18	5
Co	7.81	.24	0.38	1.35	0.02	1.11	......	......	.....	0.36	+0.60	+ .24	2
Ni	7.64	.24	0.38	1.35	0.02	1.11	......	......	.....	0.36	+0.58	+ .22	3
Cu	7.69	.19	0.44	1.17	0.03	0.98	......	......	.....	0.41	+0.59	+ .18	1
Zn	9.36	.82	0.07	2.52	0.00	1.70	......	......	.....	0.07	−0.05	− .12	1
Sr	5.65	.00	2.46	0.02	1.44	0.02	+ .10	.12	1	1.02	+1.16	+ .14	1
Y	6.5	.01	1.75	0.12	0.60	0.11	− .07	.04	2	1.15	+0.98	− .17	1
Mo	7.35	.05	0.95	0.59	0.13	0.54	......	......	.....	0.82	+1.16	+ .34	½
Ru	7.7	− .07	0.80	0.76	0.08	0.69	......	......	.....	0.72	+1.09	+0.37	½
Ba	5.19	.00	2.95	0.00	2.01	0.00	+ .43	.43	1	0.94	......	......	.....
La	5.5	0.00	−2.69	−0.01	−1.70	−0.01	+0.03	+0.04	1	+0.99	......	......	.....

TABLE XIII

El.	I.P.	Disk		Spot		$\log \frac{x'}{x}$	$\log \frac{1-x'}{1-x}$	Y_0 Eq. (22)
		$\log x$	$\log (1-x)$	$\log x'$	$\log (1-x')$			
H	13.54	−4.73	0.00	−7.27	0.00	−2.54	0.00	+0.23
Li	5.36	0.00	−2.45	0.02	−1.47	0.02	+0.98	1.21
C	11.22	2.56	0.00	4.69	0.00	2.13	0.00	0.23
O	13.56	4.79	0.00	7.35	0.00	2.56	0.00	0.23
Al	5.95	0.01	1.77	0.09	0.73	0.08	1.04	1.27
Rb	4.13	0.00	3.53	0.00	2.77	0.00	0.76	0.99
Zr	6.92	0.02	1.28	0.32	0.29	0.30	0.99	1.22
In	5.76	−0.01	−1.93	−0.06	−0.90	−0.05	+1.03	+1.26

determinations, $T' = 4720°$, $P'_e/P_e = 0.60$, and $N'/N = 1.70$, furnish quantitative evidence regarding ionization in the sun-spot spectrum. They also explain some facts which previously have been very puzzling. For example, *Ba* is completely ionized in both disk and spot, and yet some Ba^+ lines are stronger in the spectrum of the spot than in that of the disk. Lines of Sr^+ behave similarly. With an

excess of material over the spot, as indicated by the value of N'/N, the increased intensity of the spot lines is fully explained.

Mr. Russell has suggested that a check may be applied to the foregoing values of temperature, pressure, and abundance of material over spots, by calculating the percentage of ionization to be expected under the assumed conditions, on the basis of the data which he gives for the composition of the solar atmosphere.[1] S_1 (Russell's notation), the whole number of ionized atoms per unit area, is known for each element. Log T (T is here the total number of atoms per unit area) has been recalculated with $I_0=8.5$, corresponding to $T=5740°$.[2] The corresponding values of T' and S' above the spot are given by

$$\log T'=\log C+\log T \tag{24}$$

and

$$\log S_1'=\log C+\log S_1+\log \frac{x'}{x}\,, \tag{25}$$

since

$$S_1=Tx \text{ and } S_1'=T'x'\,. \tag{26}$$

By summing the values of S_1, T, etc. (not the logarithms), for all elements, the "mean ionization" for disk and spot, $\bar{x}$ and $\bar{x}'$, respectively, may be defined by the equations

$$\bar{x}=\frac{\Sigma S_1}{\Sigma T}\,, \tag{27}$$

$$\bar{x}'=\frac{\Sigma S_1'}{\Sigma T'}=\frac{\Sigma\left(S_1\frac{x'}{x}\right)}{\Sigma T}\,. \tag{28}$$

From data given in Table XIV it is found that for the metallic elements, including *Si*, $\bar{x}=0.84$, $\bar{x}'=0.33$. If, however, the permanent gases, which are not perceptibly ionized, are taken into account,

[1] *Mt. Wilson Contr.*, No. 383, p. 46; *Astrophysical Journal*, **70**, 56, 1929.

[2] Russell, *Mt. Wilson Contr.*, No. 383, p. 44; *Astrophysical Journal*, **70**, 54, 1929, and unpublished material.

ΣS_1 remains practically unchanged, while ΣT is increased by a factor of 65 (according to Russell)[1] and the resulting values are

$$\bar{x}=0.0129\,, \qquad \bar{x}'=0.0051\,.$$

Let p be the total pressure and P_e the electron pressure at the photospheric level over the disk. Then

$$P_e=\frac{p\bar{x}}{1+\bar{x}}\,. \tag{29}$$

Similarly, for the spot

$$P'_e=\frac{p'\bar{x}'}{1+\bar{x}'}\,. \tag{30}$$

Since the total pressure is proportional to the weight of the overlying material,

$$\frac{p'}{p}=\frac{N'}{N}\,. \tag{31}$$

From (29), (30), and (31),

$$\log\frac{P'_e}{P_e}=\log\frac{N'}{N}+\log\frac{\bar{x}'}{1+\bar{x}'}-\log\frac{\bar{x}}{1+\bar{x}}=\log C+\log\frac{\bar{x}'}{\bar{x}}-\log\frac{1+\bar{x}'}{1+\bar{x}}\,. \tag{32}$$

With $\log C=+0.23$, $\log(\bar{x}'/\bar{x})=-0.40$ (from [27] and [28]), and $\log[(1+\bar{x}')/(1+\bar{x})]=0.00$, $\log(P'_e/P_e)=-0.17$. This agrees with the observed value -0.22 within the errors of the determination, and is practically independent of the assumed amount of H and other elements difficult to ionize in the atmosphere so long as it is large enough to make $(1+\bar{x}')/(1+\bar{x})$ approximately unity. The accordance is strong evidence in favor of the correctness of the relatively high spot temperature, 4720°.

When Adams and Russell used this method to find the temperatures of several stars,[2] the plots of Y against E_x for a long series of Fe lines gave curves instead of straight lines, which showed that $\Delta Y/\Delta E_x$ diminishes as E_x increases. They supposed that this indicated a departure from thermodynamic equilibrium and applied an

[1] *Mt. Wilson Contr.*, No. 383, p. 63; *Astrophysical Journal*, **70**, 73, 1929.

[2] *Mt. Wilson Contr.*, No. 359; *Astrophysical Journal*, **68**, 9, 1928.

empirical correction based on the assumption that Y is a linear function not of E_x but of X, where

$$X = 1.05E_x - 0.087E_x^2 \tag{33}$$

and

$$\frac{\Delta Y}{\Delta E_x} = 1.05S'' . \tag{34}$$

Then

$$5040\left(\frac{1}{T} - \frac{1}{T'}\right) = 1.05S'' . \tag{35}$$

The sun-spot data in Table VIII, together with the empirical correction, then give $S'' = -0.281$ and $T' = 4290°$. Corresponding to this value of T', $\log (P_e''/P_e) = -1.09 \pm 0.10$. The level of ionization

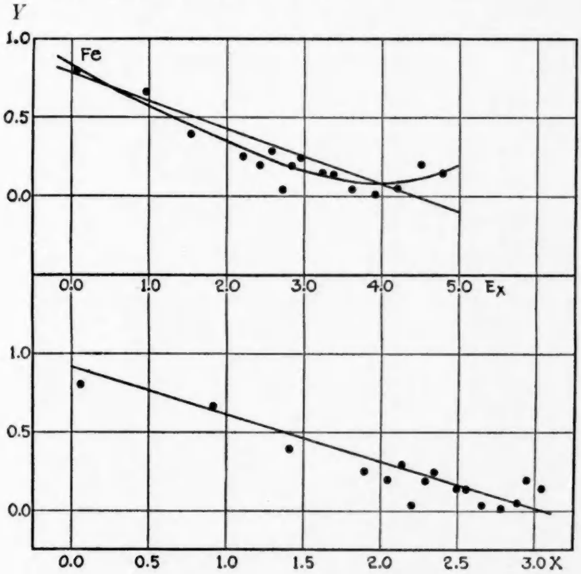


FIG. 1.—Slopes for *Fe* without (above) and with empirical correction

I_0' remains unchanged, as is to be expected, since the elements used in the calculation of I_0' have ionization potentials both less and greater than 7 volts; those with I less than 7.0 are known from observation to be more than half-ionized, and those with I greater than 7.0 to be less than half-ionized. Further, $\log C = +0.31 \pm 0.06$,

and from (27) and (28), $\bar{x}''$ now becomes 0.31 for the metals alone, or 0.0048 with the permanent gases included. The computed value of log (P_e''/P_e) is then -0.11 as compared with -1.09 derived from observation.

Plots of the *Fe* data, without and with the empirical correction, are given in Figure 1. The upper plot, which is that of Y against E_x, shows an unexplained curvature similar to that obtained in the case of the stars. Below is a plot of Y against X; the curvature has been removed, but the great discordance in log (P_e''/P_e) indicates decisively that the empirical correction is not applicable in the case of the sun-spot spectrum.

A second check suggested by Mr. Russell may be applied by using the formula for the opacity coefficient, κ, recently derived by J. A. Gaunt[1] on the basis of the quantum theory. For a given wave-length this formula states that, approximately,

$$\kappa \propto \frac{P_e}{T^{3/2}} . \tag{36}$$

If τ denotes the integrated opacity to the depth above which there are N atoms per square centimeter,

$$\tau \propto N\kappa \propto N \frac{P_e}{T^{3/2}} . \tag{37}$$

Similarly, for the spot,

$$\tau' \propto N'\kappa' \propto N' \frac{P_e'}{T'^{3/2}} . \tag{38}$$

Since in both disk and spot one "sees down" as far as possible, the opacities τ and τ' should be equal. Therefore

$$\frac{\tau'}{\tau} = \frac{N'}{N} \frac{P_e'}{P_e} \left(\frac{T'}{T}\right)^{-3/2} = 1 . \tag{39}$$

[1] *Proceedings of the Royal Society*, A, **126**, 660, 1930.

The observed values give

$$\begin{aligned} \log \frac{N'}{N} &= +0.23 \pm 0.03 \\ \log \frac{P'_e}{P_e} &= -0.22 \pm 0.07 \\ -\tfrac{3}{2} \log \frac{T'}{T} &= +0.13 \pm 0.00 \\ \hline \text{Sum} &= +0.14 \pm 0.08 \,. \end{aligned}$$

Since the theoretical value is 0.00, the agreement is tolerable.

Equation (39) may be employed to calculate theoretical values of $\log (P'_e/P_e)$ and $\log (N'/N)$ for any given value of T'/T (T being always 5740°), on the assumption that the composition of the atmosphere is unaltered. From equation (32), since, approximately, $\log [(1+\bar{x}')/(1+\bar{x})]=0$,

$$\frac{P'_e}{P_e}=\frac{N'}{N}\frac{\bar{x}'}{\bar{x}}\,, \tag{40}$$

whence from (39)

$$\left(\frac{N'}{N}\right)^2 \frac{\bar{x}'}{\bar{x}}\left(\frac{T'}{T}\right)^{-3/2}=1\,. \tag{41}$$

Therefore

$$\frac{N'}{N}=\left(\frac{\bar{x}}{\bar{x}'}\right)^{1/2}\left(\frac{T'}{T}\right)^{3/4}\,, \tag{42}$$

and from (40)

$$\frac{P'_e}{P_e}=\left(\frac{\bar{x}'}{\bar{x}}\right)^{1/2}\left(\frac{T'}{T}\right)^{3/4}\,. \tag{43}$$

Since $\bar{x}'/\bar{x}$ depends on P'_e/P_e, equation (43) must be solved by trial and error. With the given value of T' and any convenient value of $\log (P'_e/P_e)$, the corresponding level of ionization I'_0 is found by (16), and the degree of ionization x', for each element, by (18). The mean ionization, $\bar{x}'$, is then found by (28), as in Table XIV. Equation (43) will not usually be satisfied by these values; but by assuming a second trial value of $\log (P'_e/P_e)$ and interpolating between the two results, a solution is easily obtained. For example, when $T'=4720^\circ$,

the observed value $\log (P'_e/P_e) = -0.22$ leads to $\log (\bar{x}'/\bar{x}) = -0.40$ (Table XIV) and

$$\log \left[\left(\frac{\bar{x}'}{\bar{x}}\right)^{1/2}\left(\frac{T'}{T}\right)^{3/4}\right] = -0.264 .$$

Repetition of the calculations with $\log (P'_e/P_e) = -0.32$ gives

$$\log \frac{\bar{x}'}{\bar{x}} = -0.364 \quad \text{and} \quad \log \left[\left(\frac{\bar{x}'}{\bar{x}}\right)^{1/2}\left(\frac{T'}{T}\right)^{3/4}\right] = -0.246 .$$

Interpolation shows that (43) is satisfied by $\log (P'_e/P_e) = -0.257$, whence $\log (\bar{x}'/\bar{x}) = -0.386$.

TABLE XIV

El.	$\log S_1$	$10^{-5}S_1$	$\log S_1 + \log \frac{x'}{x}$	$10^{-5}\left(S_1 \frac{x'}{x}\right)$	$\log T$	$10^{-5}T$
H	5.7	5	3.1	0	[10.46]	....
Na	7.2	159	7.19	155	7.20	159
Mg	7.7	500	7.03	107	7.77	589
Al	6.4	25	6.32	21	6.41	26
Si	7.0	100	5.81	6	7.25	178
K	6.8	63	6.80	63	6.80	63
Ca	6.7	50	6.66	46	6.71	51
Ti	5.2	2	5.02	1	5.21	2
V	5.0	1	4.74	0	5.02	1
Cr	5.7	5	5.43	3	5.72	5
Mn	5.8	6	5.14	1	5.87	7
Fe	7.1	126	6.21	16	7.21	162
Co	5.4	3	4.29	0	5.64	4
Ni	5.7	5	4.59	0	5.94	9
Cu	4.9	1	3.92	0	5.09	1
Total	....	1051	....	419	....	1257

These calculations have been made for three temperatures lower than that of the photosphere, with the results given in Table XV. The values of $\log (P'_e/P_e)$ were obtained by the method just described and used to compute the next three columns by (42), (43), and (16), respectively. The last two columns give the values of $\log (P'_e/P_e)$ and $\log (N'/N)$ derived from observation: for the disk, which was taken as standard, and for the spot on two different assumptions re-

garding temperature. The spot temperature 4720° gives good agreement between the observed and the theoretical values of pressure. This is *not* true for the lower temperature 4290°. These results indicate that the difference in temperature between disk and spot explains completely the variation of the intensities of the lines in the two spectra.

With the adopted values of temperature, pressure, and abundance derived for the sun-spot spectrum, the way is open to calculate the intensities of spot lines from formulae previously given. This has been done for a few of the most interesting elements, *H*, *Li*, *C*, *O*,

TABLE XV

T'	$\log \frac{T'}{T}$	$\log \frac{P'_e}{P_e}$	$\log \frac{N'}{N}$	$\log \bar{x}'$	I'_0	Observed	
						$\log \frac{P'_e}{P_e}$	$\log \frac{N'}{N}$
4290°	−0.126	−0.375	+0.187	−0.642	6.40	−1.09	+0.31
4720	.086	.257	.129	.466	7.02	−0.22	+0.23
5200	.042	.112	.047	.238	7.70	....	....
5740	−0.000	−0.000	+0.000	−0.080	8.50	0.00	0.00

Mg, *Si*, *Rb*, and *In*. Tables XII and XIII contain the data for obtaining Y_0, which is to be found from (22) with the adopted value of $\log C$.[1] Equation (8) then gives the calculated value of Y, namely,

$$Y_c = Y_0 + S\, E_x\,. \tag{44}$$

Since from (5)

$$\left.\begin{aligned} \log N'_x &= B \log A'\,, \\ \log N_x &= B \log A\,, \end{aligned}\right\} \tag{45}$$

and

$$Y_c = \log \frac{N'_x}{N_x} = B \log \frac{A'}{A}\,, \tag{46}$$

$$\log \frac{A'}{A} = \frac{Y_c}{B}\,. \tag{47}$$

[1] Values of Y_0 in Table XIII were found in this way.

For a known disk intensity, log A can be read from the calibration table.[1] B is known. Hence log A' can then be found from (47) and the corresponding calculated spot intensity read from the table.

An example will best illustrate the method. The disk intensity of $H\gamma$ is 20. Log A for a line of this intensity is 3.72. Y_0 for H is $+0.23$ (Table XIII). $S=-0.19$, E_x for $H\gamma$ is 10.155 volts, and Y_c from (44) is -1.70. The value of B from the calibration table for λ 4340 is 1.092. From (47), log $(A'/A)=-1.55$. Therefore log A' equals 2.17, which gives a calculated spot intensity of 4. The estimated spot intensity of $H\gamma$ is 5N? .

Aside from the strongest H lines, $H\alpha$ and $H\beta$, for which estimates are valueless, the behavior of the H lines in the spot spectrum is fully accounted for by spot intensities thus calculated. The same is true of the C, O, Si, Si^+, Mg, and Mg^+ lines. The average residual O – C for these lines is ± 1.0 Rowland unit, with the omission of $H\beta$ and the Mg triplet, which are too strong for reliable estimates.

For Li, Rb, and In, disk intensities of lines have been calculated from the observed spot intensities. The Li and Rb lines should have intensities -1 and -2 in the disk, and the In line at λ 4511 should be just too faint to be seen, its calculated disk intensity being -3.

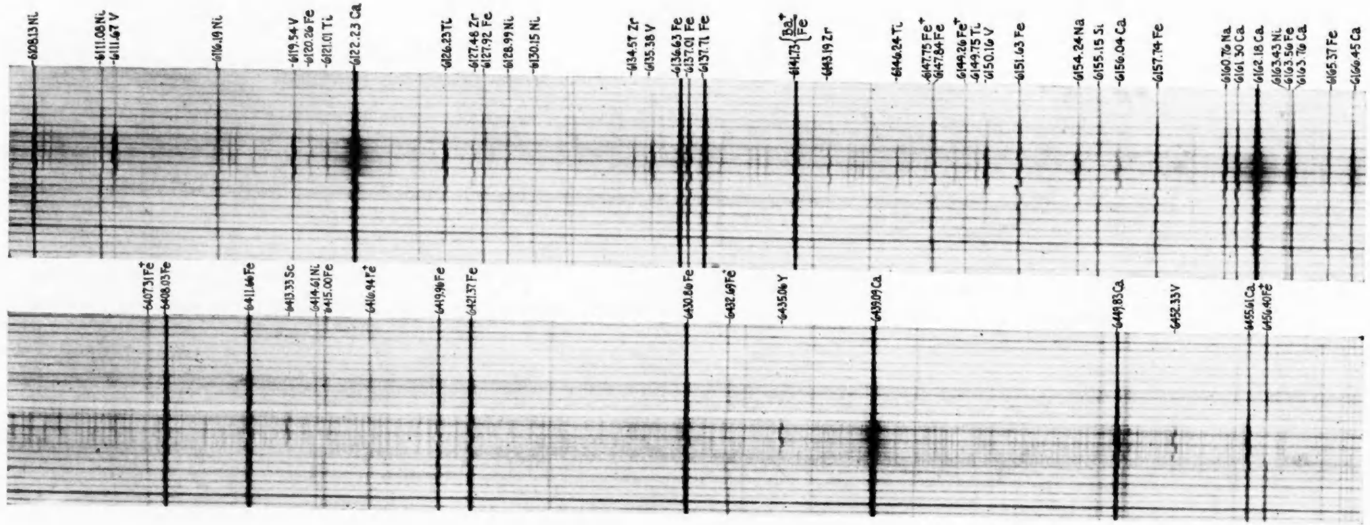
Table XVI gives the observed and calculated spot intensities for a number of the most extreme cases, including data for H, C, O, Si, Si^+, Mg, and Mg^+, the elements which contribute the lines of highest excitation potential in the spot spectrum. Table XVII gives similar data for all available[2] lines in the regions λλ 6108–6166 and λλ 6407–6456 (Plate XV), within which occur many that are conspicuously affected in the spot spectrum.

The discrepancies are greatest for the strongest and the weakest lines, for several reasons. Estimated intensities are of very little value for the strongest lines, such as the Mg triplet, D_1 and D_2, $H\beta$ and $H\alpha$, and above all for H and K. These lines can be handled by microphotometric curves. Further, the Rowland calibration-curve is least reliable for the faintest and the strongest intensities. Finally, it is well known

[1] Russell, Adams, and Moore, *Mt. Wilson Contr.*, No. 358; *Astrophysical Journal*, **68**, 1, 1928.

[2] In calculating spot intensities it is necessary to know the excitation potential of the line and the ionization potential of the element to which the line is attributed. Blends cannot be considered.

PLATE XV



RED REGION OF SUN-SPOT SPECTRUM

that very faint solar lines often show unusual strengthening in the spot spectrum. By calculating spot intensities for all the elements, it may be possible to ascertain the amount and cause of the exceptional

TABLE XVI

λ I.A.	El.	Observed Intensity		E.P.	$\log \frac{A'}{A}$	Calc. Spot Intensity
		Disk	Spot			
3970.078....	$H\epsilon$	5N?	1N?	10.155	−1.47	2
4101.750....	$H\delta$	15N?	3?	10.155	−1.50	4
4340.477....	$H\gamma$	20N	5N?	10.155	−1.55	4
4861.344....	$H\beta$	30	12?	10.155	−1.67	5
4770.001....	*C*	−2	ob?	7.450	−1.16	<−3
4775.886....	*C*	−2	ob?	7.455	−1.16	<−3
7771.954....	*O*	2	−2	9.106	−1.97*	−2
7774.177....	*O*	2	−2	9.106	−1.97*	−2
7775.394....	*O*	1	−3	9.106	−1.97*	−2
3905.534....	*Si*	8	8	1.900	+0.18	10
4102.945....	*Si*	5	5N	1.900	+0.19	6
5645.621....	*Si*	1	−1N	4.908	−0.39	0
5665.566....	*Si*	1N	−1N	4.899	−0.39	0
5690.435....	*Si*	3	1	4.908	−0.39	2
5701.111....	*Si*	1N	−1	4.908	−0.39	0
5708.408....	*Si*	3N	1	4.932	−0.40	2
5754.240....	*Si*	−1	−2	4.932	−0.40	−2
5772.152....	*Si*	3	2	4.899	−0.39	2
5793.083....	*Si*	3	1N	4.908	−0.39	2
5797.869....	*Si*	3	−1N	4.932	−0.41	2
5948.552....	*Si*	6	4	5.060	−0.44	5
6347.104....	Si^+	2N	ob	8.085	−2.92	−3
6371.362....	Si^+	1Nd?	ob	8.085	−2.93	<−3
3938.411....	*Mg*	3	3	4.327	+0.15	3
3986.761....	*Mg*	6	6N	4.327	+0.15	7
4057.520....	*Mg*	5	5	4.327	+0.15	6
4167.279....	*Mg*	8	7	4.327	+0.15	9
4351.923....	*Mg*	5Nd?	4	4.327	+0.16	6
4571.104....	*Mg*	5	7	0.000	+0.94	11
4703.005....	*Mg*	10	10	4.327	+0.16	12
5167.330....	*Mg*	15	16	2.697	+0.49	35
5172.700....	*Mg*	20	22	2.700	+0.49	>40
5183.621....	*Mg*	30	30	2.705	+0.49	>40
5528.420....	*Mg*	8	8	4.327	+0.18	10
5711.098....	*Mg*	6	6	4.327	+0.18	7
4481.142....	Mg^+	0	−2	8.825	−1.99	<−3
4481.340....	Mg^+	0	ob	8.826	−1.99	<−3

* The value of $B=0.76$ for *O* was extrapolated from the calibration table.

strengthening of the faint lines. A check on the calibration-curve for the fainter Rowland intensities may perhaps also be made.

TABLE XVII

λ I.A.	El.	Intensity		E.P.	$\log\frac{A'}{A}$	Calc. Spot Intensity
		Disk	Spot			
6108.130....	*Ni*	6	8	1.669	+0.31	8
6111.085....	*Ni*	2	1	4.071	−0.21	2
6111.668....	*V*	0d?	6	1.039	+1.21	2
6119.537....	*V*	1	8	1.059	+1.21	3
6120.257....	*Fe*	−1	1	0.911	+0.74	0
6121.012....	*Ti*	−2	2	1.871	+1.09	0
6122.231....	*Ca*	10	22?	1.878	+1.09	>40
6126.230....	*Ti*	1	8	1.062	+1.28	4
6127.481....	*Zr*	−3N	3	0.153	+1.35	−1
6127.918....	*Fe*	3	3	2.269	+0.44	4
6128.990....	*Ni*	1	2	1.669	+0.31	2
6130.147....	*Ni*	1	0	4.248	−0.25	1
6135.378....	*V*	−1N	6	1.046	+1.21	1
6136.631....	*Fe*	8	10	2.443	+0.41	13
6137.009....	*Fe*	3	4	2.188	+0.46	4
6137.709....	*Fe*	7	9	2.577	+0.38	10
6143.189....	*Zr*	−3N	3	0.070	+1.38	−1
6146.244....	*Ti*	−2	2	1.865	+1.11	0
6147.749....	*Fe*$^+$	2	0b	3.872	−1.60	−1
6149.255....	*Fe*$^+$	2	−2	3.873	−1.60	−1
6149.748....	*Ti*	−3N	1	2.151	+1.04	−1
6150.158....	*V*	0Nd?	7	0.299	+1.37	3
6151.630....	*Fe*	4	5	2.167	+0.47	5
6154.235....	*Na*	2	6	2.093	+0.87	4
6156.037....	*Ca*	−1	3	2.510	+0.96	1
6160.759....	*Na*	3	7	2.095	+0.87	5
6161.302....	*Ca*	4	8	2.512	+0.96	8
6162.185....	*Ca*	15	25?	1.891	+1.10	>40
6163.426....	*Ni*	1	1?	4.088	−0.22	1
6163.561....	*Fe*	1	1	2.188	+0.46	2
6163.760....	*Ca*	3	8	2.510	+0.96	5
6166.446....	*Ca*	5	7	2.510	+0.96	12
6407.310....	*Fe*$^+$	0N	0b	3.872	−1.64	−3
6408.033....	*Fe*	5	7	3.671	+0.14	5
6411.665....	*Fe*	7	8	3.638	+0.15	8
6414.608....	*Ni*	0	−2	4.136	−0.24	0
6416.935....	*Fe*$^+$	1	−1	3.875	−1.65	−2
6419.964....	*Fe*	4	3	4.713	−0.09	4
6421.367....	*Fe*	7	8	2.269	+0.46	11
6430.863....	*Fe*	7	9	2.167	+0.48	12
6432.690....	*Fe*$^+$	1	−2	2.879	−1.43	−2
6435.056....	*Y*	−3	3	0.065	+1.61	0
6439.090....	*Ca*	8	11	2.515	+0.99	30
6449.827....	*Ca*	6	9	2.510	+0.99	18
6452.331....	*V*	−1N	2	1.190	+1.21	1
6455.613....	*Ca*	2	6	2.512	+0.99	4
6456.396....	*Fe*$^+$	3	−1	3.887	−1.65	0

The agreement of observed and calculated spot intensities for the lines of highest excitation potential, in the region where the calibration-curve is best known, indicates that the calculated intensities may be used with some assurance for all the elements in the spot spectrum. This possibility may lead to the solution of some puzzling questions which still remain unanswered.

All the numerous and important differences between the spectra of sun-spots and of the disk, as regards the intensities of lines of atomic origin, are fully explained by the difference in the effective temperatures of spot and disk. The diminished ionization results in a greater transparency of the material. The reversing layer is therefore thicker over the spots, but the electron pressure is lower.

6. OTHER ASPECTS OF THE PROBLEM

The field of study touched upon in the present discussion offers many interesting aspects which cannot be considered here. The work on the infra-red solar spectrum has already been mentioned. When more spectra have been analyzed and when improved wave-lengths have been obtained, greater definiteness can be given to the identification of many solar and spot lines. A detailed and accurate photometric study of the solar lines, now being carried out at Mount Wilson, is needed to correct the present intensities and establish an intensity scale which is uniform over the whole range of the solar spectrum. The band lines, and the whole question of the presence of molecules in the sun and spots, lie outside the field of the present investigation. A study of the Zeeman effects of lines in the spot spectrum and their possible relation to the unequal strengthening of different lines is most inviting and may be attempted in the near future. Many other problems equally interesting might be mentioned. The present work has been done with the hope that it will be useful in the solution of some of them.

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TABLE XVIII*

REFERENCES TO SPECTROSCOPIC MATERIAL

(Tabular numbers refer to bibliography)

El.	Neutral	Ionized	El.	Neutral	Ionized
B	20, 24	....	*La*	....	14, 22
C	7	....	*Pr*	....	27
N	2, 7, 28	....	*Nd*	....	27
O	6, 28	....	*Sa*	....	27
Al	12	....	*Eu*	....	27
Si	5	....	*Gd*	....	27
S	28	....	*Tb*	....	27
V	14	....	*Dy*	....	27
Cr	9	9	*Er*	....	27
Mn	3	....	*Yb*	....	27
Fe	1, 22	....	*Lu*	....	16, 18, 27
Co	22	4	*Hf*	....	15
Ni	23	....	*W*	13	....
Zr	11	10	*Re*	17, 19	....
Mo	....	8	*Os*	28	....
Pd	25	....	*Ir*	28	....
Cd	26	....	*Pt*	28	....
Sb	21, 28	....	*Tl*	28	....

* For other lists of references see the "Revised Rowland Table," *Carnegie Institution of Washington Publication*, No. 396; *Papers of the Mount Wilson Observatory*, **3**, 237, 1928, and *Mount Wilson Contribution*, No. 383, p. 70; *Astrophysical Journal*, **70**, 80, 1929.

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MEAN IONIZATION IN STELLAR ATMOSPHERES[1]

By HENRY NORRIS RUSSELL[2]

ABSTRACT

The mean ionization is calculated for a gas of the composition found by the writer for the solar atmosphere, over a range of temperature from 1700° to 25,000° K, and of electron pressure from 10^3 to 10^{-20} atm. Tables are given for the metals alone, for hydrogen, and for the assumed mixture of the two, with the reciprocal temperature and electron pressure as arguments, and again with the gas pressure as second argument. Diagrams illustrate the effects of the successive ionizations of the constituents.

1. In investigations of stellar atmospheres the ratio of the electron pressure to the total gas pressure at any point is of importance. This ratio varies greatly with the composition of the gas, and most investigators have adopted, in numerical applications, greatly simplified and rather arbitrary assumptions regarding the latter. For example, E. A. Milne,[3] while discussing the luminosities of the stars, adopts, for the whole atmosphere, ionization potentials equal to those of calcium, remarking:

> Of course this does not mean that we are assuming the photospheric layers to be entirely composed of calcium. But it is particularly desirable to avoid the appearance of choosing I.P.'s to suit the later results. All we want are some ionization potentials that determine the abundance of free electrons.

Professor Milne's caution is well founded. But the assumption that a star's atmosphere is similar in composition to the sun's is equally impartial, and very much more likely to be near the truth.[4]

It has seemed worth while, therefore, to calculate in detail the relation between the total and the electron pressures for a mass of gas having substantially the composition which the writer recently found for the sun's atmosphere.[5]

The outstanding uncertainties in composition, which are in some

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 447.

[2] Research Associate of the Mount Wilson Observatory, Carnegie Institution of Washington.

[3] *Monthly Notices of the Royal Astronomical Society*, **90**, 35, 1929.

[4] C. H. Payne, *Stellar Atmospheres* (1925), p. 189.

[5] *Mt. Wilson Contr.*, No. 383; *Astrophysical Journal*, **70**, 56 and 73, 1929.

cases large, have fortunately very little influence on the present problem. At low temperatures almost all the electrons come from the metals; and the relative proportions of these (including *Si*) are well determined from the spectroscopic data. Hydrogen is so abundant that, as soon as it begins to be considerably ionized, it furnishes the great majority of the electrons. Oxygen has almost exactly the same I.P. as hydrogen and, for the present purpose, can be counted with it. The other light elements are present in proportions too small to affect the total sensibly, and the same is true of the electrons coming from the second ionization of the metals.

The writer's estimate makes hydrogen sixty times as abundant (atom for atom) as all the metals together. A. Unsöld,[1] from quite independent data, makes the ratio 40. Second ionization of all the metallic atoms would therefore affect the electron pressure by only 2 per cent when once the hydrogen was ionized.

It suffices, therefore, to consider only the first ionization of hydrogen and of the metals, and only a few of the latter are present in sufficient proportions to contribute sensibly to the total supply of electrons. The composition which has been adopted as typical is as given in Table I, the total number of metallic atoms being taken as one thousand.

There would be no advantage in including the less abundant constituents—indeed, chromium might have been omitted without sensible loss of accuracy. Since the ratio of hydrogen to the metals is still uncertain, the two have been kept separate until the end of the computations.

2. For the ionization of a gas in thermodynamic equilibrium at temperature T and electron pressure P_e, Saha's formula, as modified by A Pannekoek,[2] runs

$$\log\left(\frac{x}{1-x}P_e\right)=-\frac{5040I}{T}+\frac{5}{2}\log T-6.49+\log\left(\frac{2B'(T)}{B(T)}\right),$$

where B and B' are the partition functions for the neutral and the ionized atoms. If, however, the gas is in a stellar atmosphere, ex-

[1] *Zeitschrift für Astrophysik*, **3**, 81, 1931.

[2] *Handbuch der Astrophysik*, **3**, Part I, 278, 1930.

posed to radiation from a photosphere of effective temperature T_1, Pannekoek[1] has shown that the second member must be increased by

$$\log\left(\frac{T_1}{2T}\right)+\frac{5040I}{T}-\frac{5040I}{T_1}\ .$$

According to A. S. Eddington,[2] T/T_1 ranges from 0.84 at the top of the atmosphere to 0.88 in the upper part of the photosphere

TABLE I

COMPOSITION OF A REPRESENTATIVE STELLAR ATMOSPHERE

Element	*K*	*Na*	*Al*	*Ca*	*Cr*	*Mg*	*Fe*	*Si*	*H*
I.P.	4.33	5.09	5.95	6.09	6.75	7.61	7.63	8.14	13.54
No. of atoms	50	126	20	40	10	470	126	158	61,500

($\tau=0.25$). Adopting $T=0.86T_1$, we then have for ionization in the atmosphere

$$\log\frac{x}{1-x}=-\frac{5040I}{T_1}+\frac{5}{2}\log T_1-6.59+\log\left(\frac{B'(T)}{B(T)}\right)-\log P_e\ . \qquad (1)$$

The fraction x of atoms which are ionized is here given in terms of the photospheric temperature and the local electron pressure. Milne's[3] investigations show that the mean value of x throughout the atmosphere above any given level is very nearly equal to that computed from (1) with an electron pressure half that at the base-level. The partition functions are given by the equation

$$B(T)=\sum g_n f_n\ ; \qquad (2)$$

where the g's are the statistical weights of the various energy states of the atom and the f's the Boltzmann factors defined by $\log f=-5040E/T$, E being the excitation potential. The list of these states given by the writer[4] is almost complete. Additional states included

[1] *Ibid.*, p. 289.

[2] *The Internal Constitution of the Stars* (1926), pp. 323, 335.

[3] *Monthly Notices of the Royal Astronomical Society*, **89**, 17, 1928; also Russell, *Mt. Wilson Contr.*, No. 383; *Astrophysical Journal*, **70**, 76, 1929.

[4] *Mt. Wilson Contr.*, No. 383; *Astrophysical Journal*, **70**, 44 ff., 1929. See also note on p. 51.

in the present discussion are for *K:* ^{2}P, $E=1.61$; ^{2}S, 2.61; ^{2}D, 2.68; for *Na:* ^{2}S, 3.17; ^{2}D, 3.60; for Al^+: ^{3}P, 4.62; for Mg^+: ^{2}P, 4.46.

The resulting values are given in Table II. These values are all negative; that is, consideration of the statistical weights diminishes the computed ionization. This happens mainly because the weight of the ground state is usually greater for the neutral atom, but the ratio of the weights of these states (which is given under $T=0$) is often an insufficient approximation to that of the partition functions.

TABLE II

PARTITION FUNCTIONS; LOG (B'/B)

T $5040/T$	0° ∞	2520° 2.0	3360° 1.5	5040° 1.0	6720° 0.75	10,080° 0.5
K	−0.30	−0.30	−0.31	−0.34	−0.39	−0.54
Na	.30	.30	.30	.31	.33	.43
Al	.48	.48	.48	.48	.48	.47
Ca	.00	.00	.00	.00	.01	.04
Cr	.07	.10	.15	.27	.32	.31
Mg	.00	.00	.00	.00	.02	.07
Fe	.22	.12	.09	.07	.07	.10
Si	.17	.18	.19	.22	.24	.27
H	−0.30	−0.30	−0.30	−0.30	−0.30	−0.30

3. For a given value of $5040/T$, log $[xP_e/(1-x)]$ was first determined for each element by (1), then x, for suitable values of P_e, and finally the "mean ionization" x_m, for the metals, by the equation

$$1000x_m=\sum Ax\ , \tag{3}$$

where the A's are the abundance factors of Table I. The results appear in Table III, which gives log $(1/x_m)$ with $5040/T$ and log P_e as arguments, P_e being measured in atmospheres.

No entries are made in the right-hand part of the table below the value 0.00, corresponding to the completion of the first ionization—higher ionization being neglected for reasons already stated. At the opposite corner no entries are made when the theoretical fraction of ionization is less than 10^{-6}. They can easily be supplied if desired, since in this region x_mP_e is constant for a given temperature. For an atmosphere of the specified composition the data are accurate even for very low ionization, but the electrons come mainly

TABLE III

Mean Ionization of Metals; Values of log $(1/x_m)$

log P_e	5040/T and T																				
	0.2 25,200°	0.3 16,800°	0.4 12,600°	0.5 10,080°	0.6 8400°	0.7 7200°	0.8 6300°	0.9 5600°	1.0 5040°	1.1 4582°	1.2 4200°	1.3 3877°	1.4 3600°	1.5 3360°	1.6 3150°	1.8 2800°	2.0 2520°	2.2 2291°	2.6 1939°	3.0 1680°	4.0 1260°
3..	0.55	1.40	2.26	3.02	3.72	4.36	4.95	5.55	6.13	...	...	...	...	...	...	...	...	...	...	...	...
2..	0.10	0.58	1.31	2.04	2.72	3.36	3.95	4.55	5.13	5.68	...	...	...	...	...	...	...	...	...	...	...
1..	0.01	0.11	0.52	1.14	1.75	2.38	2.95	3.55	4.13	4.68	5.22	5.73	...	...	...	...	...	...	...	...	...
0..	0.00	0.01	0.11	0.46	0.86	1.44	1.99	2.56	3.13	3.68	4.22	4.73	5.28	5.81	...	...	...	...	...	...	...
−1..	...	0.00	0.01	0.11	0.42	0.74	1.17	1.64	2.15	2.68	3.22	3.73	4.28	4.81	5.32	...	...	...	...	...	...
2..	...	...	0.00	0.01	0.10	0.37	0.68	0.97	1.32	1.75	2.25	2.73	3.28	3.81	4.32	5.33	...	...	...	...	...
3..	...	...	...	0.00	0.01	0.09	0.32	0.60	0.85	1.09	1.46	1.84	2.31	2.81	3.32	4.33	5.31	...	...	...	...
4..	...	...	...	...	0.00	0.01	0.07	0.29	0.56	0.74	0.90	1.17	1.49	1.88	2.35	3.33	4.31	5.30	...	...	...
5..	...	...	...	...	...	0.00	0.01	0.06	0.25	0.53	0.68	0.81	1.00	1.24	1.53	2.35	3.31	4.30	...	...	...
6..	...	...	...	...	...	...	0.00	0.01	0.05	0.23	0.48	0.63	0.73	0.86	1.05	1.55	2.34	3.30	5.25	...	...
7..	...	...	...	...	...	...	...	0.00	0.01	0.04	0.19	0.43	0.61	0.69	0.79	1.10	1.56	2.36	4.25	...	...
8..	...	...	...	...	...	...	...	...	0.00	0.00	0.04	0.14	0.40	0.57	0.66	0.83	1.14	1.57	3.25	5.11	...
9..	...	...	...	...	...	...	...	...	...	...	0.00	0.02	0.12	0.33	0.54	0.69	0.84	1.17	2.27	4.11	...
10..	...	...	...	...	...	...	...	...	...	...	...	0.00	0.02	0.09	0.29	0.61	0.72	0.87	1.53	3.11	...
11..	...	...	...	...	...	...	...	...	...	...	...	...	0.00	0.02	0.08	0.47	0.64	0.74	1.23	2.17	...
12..	...	...	...	...	...	...	...	...	...	...	...	...	...	0.00	0.01	0.20	0.57	0.66	0.92	1.50	5.76
13..	...	...	...	...	...	...	...	...	...	...	...	...	...	...	0.00	0.04	0.38	0.62	0.78	1.24	4.76
14..	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	0.00	0.13	0.55	0.70	0.86	3.76
15..	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	0.03	0.27	0.64	0.79	2.76
16..	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	0.00	0.09	0.61	0.74	1.88
17..	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	0.01	0.56	0.67	1.41
18..	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	0.00	0.39	0.63	1.27
19..	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	0.12	0.61	1.09
−20..	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	0.03	0.57	0.83

TABLE IV

IONIZATION OF HYDROGEN; VALUES OF LOG $(1/x_h)$

LOG P_e	5040/T AND T															
	0.2 25,200°	0.3 16,800°	0.4 12,600°	0.5 10,080°	0.6 8400°	0.7 7200°	0.8 6300°	0.9 5600°	1.0 5040°	1.1 4582°	1.2 4200°	1.3 3877°	1.4 3600°	1.5 3360°	1.6 3150°	1.8 2800°
3	1.60	3.38	5.06	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
2	0.69	2.38	4.06	5.66	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
1	0.14	1.40	3.06	4.66	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
0	0.02	0.53	2.06	3.66	5.20	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
−1	0.00	0.10	1.10	2.66	4.20	5.69	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
2	.....	0.01	0.33	1.67	3.20	4.69	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
3	.....	0.00	0.05	0.74	2.20	3.69	5.21	.....	.....	.....	.....	.....	.....	.....	.....	.....
4	.....	.....	0.00	0.16	1.23	2.69	4.21	5.70	.....	.....	.....	.....	.....	.....	.....	.....
5	.....	.....	.....	0.02	0.41	1.70	3.21	4.70	.....	.....	.....	.....	.....	.....	.....	.....
6	.....	.....	.....	0.00	0.06	0.77	2.21	3.70	5.17	.....	.....	.....	.....	.....	.....	.....
7	.....	.....	.....	.....	0.01	0.17	1.24	2.70	4.17	5.62	.....	.....	.....	.....	.....	.....
8	.....	.....	.....	.....	0.00	0.02	0.42	1.71	3.17	4.62	.....	.....	.....	.....	.....	.....
9	.....	.....	.....	.....	.....	0.00	0.07	0.78	2.17	3.62	5.08	.....	.....	.....	.....	.....
10	.....	.....	.....	.....	.....	.....	0.01	0.18	1.20	2.62	4.08	5.52	.....	.....	.....	.....
11	.....	.....	.....	.....	.....	.....	0.00	0.02	0.39	1.63	3.08	4.52	5.96	.....	.....	.....
12	.....	.....	.....	.....	.....	.....	.....	0.00	0.06	0.71	2.08	3.52	4.96	.....	.....	.....
13	.....	.....	.....	.....	.....	.....	.....	.....	0.01	0.15	1.12	2.52	3.96	5.38	.....	.....
14	.....	.....	.....	.....	.....	.....	.....	.....	0.00	0.02	0.34	1.53	2.96	4.38	5.80	.....
15	.....	.....	.....	.....	.....	.....	.....	.....	.....	0.00	0.05	0.63	1.96	3.38	4.80	.....
16	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0.00	0.12	1.00	2.38	3.80	.....
17	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0.01	0.28	1.40	2.80	5.64
18	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0.00	0.04	0.53	1.81	4.64
19	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0.00	0.10	0.86	3.64
−20	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0.01	0.21	2.64

TABLE V

MEAN IONIZATION; HYDROGEN *plus* METALS; VALUES OF LOG (1/X)

LOG P_e	5040/T AND T																				
	0.2 25,200°	0.3 16,800°	0.4 12,600°	0.5 10,080°	0.6 8400°	0.7 7200°	0.8 6300°	0.9 5600°	1.0 5040°	1.1 4582°	1.2 4200°	1.3 3877°	1.4 3600°	1.5 3360°	1.6 3150°	1.8 2800°	2.0 2520°	2.2 2291°	2.6 1939°	3.0 1680°	4.0 1260°
3	1.54	2.99	4.02	4.81	5.52	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
2	0.67	2.08	3.06	3.83	4.52	5.16	5.75	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1	0.14	1.29	2.25	2.93	3.55	4.18	4.75	5.35	5.93	...	...	...	...	...	...	...	...	...	...	...	...
0	0.02	0.52	1.68	2.24	2.66	3.24	3.79	4.36	4.93	5.48	...	...	...	...	...	...	...	...	...	...	...
−1	0.00	0.10	1.03	1.84	2.22	2.54	2.97	3.44	3.95	4.48	5.02	5.53	...	...	...	...	...	...	...	...	...
2	...	0.01	0.32	1.44	1.88	2.17	2.48	2.77	3.12	3.55	4.35	4.53	5.08	5.61	...	...	...	...	...	...	...
3	...	0.00	0.05	0.71	1.67	1.88	2.12	2.40	2.65	2.89	3.26	3.64	4.11	4.61	5.12	...	...	...	...	...	...
4	...	...	0.00	0.16	1.13	1.76	1.87	2.09	2.36	2.54	2.70	2.97	3.29	3.68	4.15	5.13	...	...	...	...	...
5	...	...	0.00	0.02	0.40	1.45	1.79	1.86	2.05	2.33	2.48	2.61	2.80	3.04	3.33	4.15	5.11	...	...	...	...
6	...	...	...	0.00	0.06	0.74	1.66	1.80	1.85	2.03	2.28	2.43	2.53	2.66	2.85	3.35	4.14	5.10	...	...	...
7	...	...	...	...	0.01	0.17	1.14	1.75	1.81	1.84	1.99	2.23	2.41	2.49	2.59	2.90	3.36	4.16	...	...	...
8	...	...	...	...	0.00	0.02	0.41	1.46	1.78	1.80	1.84	1.94	2.20	2.37	2.46	2.63	2.94	3.37	5.05	...	...
9	...	...	...	...	0.00	0.00	0.07	0.75	1.65	1.79	1.80	1.82	1.92	2.13	2.34	2.49	2.64	2.97	4.07	5.91	...
10	...	...	...	...	...	...	0.01	0.19	1.11	1.75	1.80	1.80	1.82	1.89	2.09	2.41	2.52	2.67	3.33	4.91	...
11	...	...	...	...	...	...	0.00	0.02	0.38	1.41	1.78	1.80	1.80	1.81	1.88	2.27	2.44	2.54	3.03	3.97	...
12	...	...	...	...	...	...	...	0.00	0.06	0.69	1.62	1.79	1.80	1.80	1.81	2.00	2.37	2.46	2.72	3.30	...
13	...	...	...	...	...	...	...	0.00	0.01	0.15	1.05	1.72	1.80	1.80	1.80	1.84	2.18	2.42	2.58	3.04	...
14	...	...	...	...	...	...	...	...	0.00	0.02	0.33	1.35	1.77	1.80	1.80	1.80	1.93	2.35	2.50	2.66	5.56
15	...	...	...	...	...	...	...	...	...	0.00	0.05	0.61	1.58	1.78	1.80	1.80	1.83	2.07	2.44	2.59	4.56
16	...	...	...	...	...	...	...	...	...	0.00	0.01	0.12	0.94	1.70	1.80	1.80	1.80	1.89	2.41	2.54	3.68
17	...	...	...	...	...	...	...	...	...	...	0.00	0.01	0.28	1.26	1.76	1.80	1.80	1.81	2.36	2.47	3.21
18	...	...	...	...	...	...	...	...	...	...	...	0.00	0.04	0.51	1.51	1.80	1.80	1.80	2.19	2.43	3.07
19	...	...	...	...	...	...	...	...	...	...	...	...	0.00	0.10	0.82	1.79	1.80	1.80	1.92	2.41	2.89
−20	...	...	...	...	...	...	...	...	...	...	...	...	...	0.01	0.21	1.74	1.80	1.80	1.83	2.37	2.63

from the potassium, and the amount of this in the solar atmosphere is relatively uncertain.

The corresponding values of log $(1/x_h)$ for hydrogen are given in Table IV.

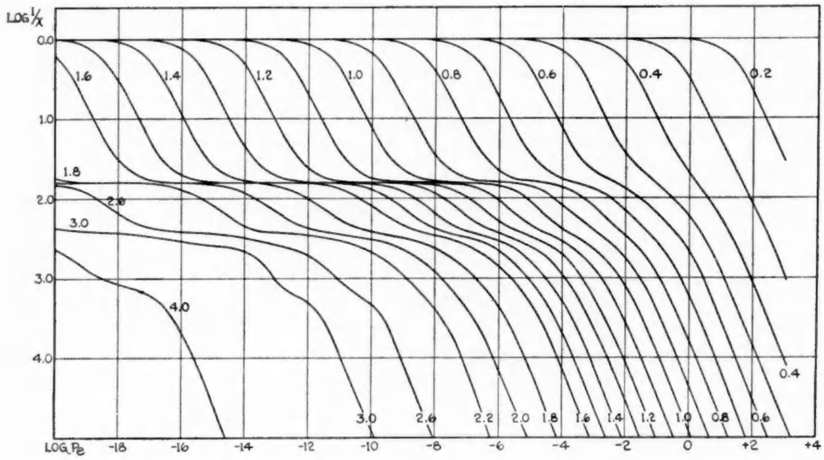

FIG. 1

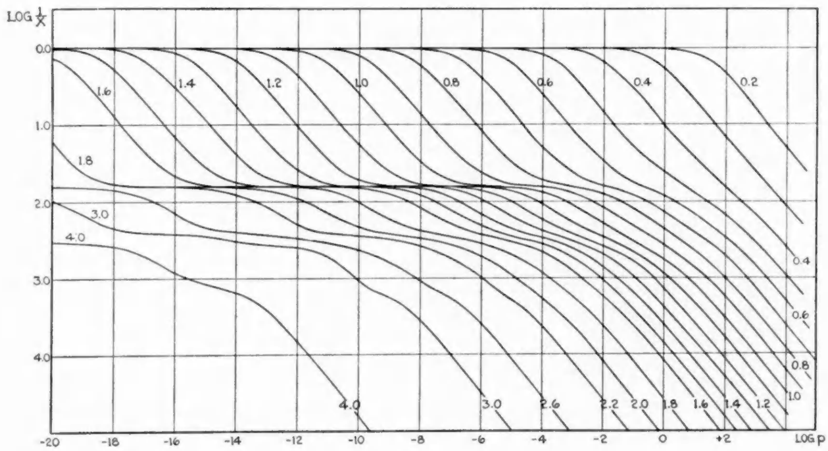

FIG. 2

For an atmosphere composed of hydrogen and metallic vapors, in the assumed proportions, the mean ionization X is given by

$$X=0.984\,x_h+0.016\,x_m\,. \tag{4}$$

The resulting values of log $(1/X)$ are given in Table V.

TABLE VI

MEAN IONIZATION FOR GIVEN GAS PRESSURE; LOG $(1/X)$ AND LOG p

LOG p	5040/T AND T																				
	0.2 25,200°	0.3 16,800°	0.4 12,600°	0.5 10,080°	0.6 8400°	0.7 7200°	0.8 6300°	0.9 5600°	1.0 5040°	1.1 4582°	1.2 4200°	1.3 3877°	1.4 3600°	1.5 3360°	1.6 3150°	1.8 2800°	2.0 2520°	2.2 2291°	2.6 1939°	3.0 1680°	4.0 1260°
4..	1.29	2.06	2.58	2.96	3.28	3.60	3.90	4.18	4.47	4.74	5.01	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
3..	0.80	1.61	2.16	2.53	2.82	3.11	3.43	3.70	3.97	4.25	4.52	4.75	.....	.....	.....	.....	.....	.....	.....	.....	.....
2..	0.33	1.15	1.80	2.16	2.45	2.69	2.99	3.23	3.50	3.76	4.03	4.28	4.55	4.79	.....	.....	.....	.....	.....	.....	.....
1..	0.08	0.68	1.43	1.88	2.16	2.40	2.63	2.85	3.07	3.31	3.59	3.81	4.06	4.30	4.56	.....	.....	.....	.....	.....	.....
0..	0.00	0.26	1.00	1.60	1.92	2.12	2.35	2.55	2.75	2.94	3.13	3.37	3.59	3.83	4.07	4.57	.....	.....	.....	.....	.....
− 1..	.....	0.05	0.53	1.29	1.75	1.91	2.09	2.31	2.51	2.65	2.77	2.98	3.18	3.40	3.60	4.07	4.56	.....	.....	.....	.....
2..	.....	0.00	0.18	0.83	1.47	1.79	1.89	2.07	2.28	2.46	2.58	2.68	2.86	3.02	3.22	3.61	4.07	4.59	.....	.....	.....
3..	.....	.....	0.02	0.36	1.07	1.62	1.80	1.88	2.04	2.27	2.42	2.52	2.61	2.73	2.90	3.23	3.60	4.09	.....	.....	.....
4..	.....	.....	0.00	0.10	0.60	1.29	1.72	1.80	1.87	2.03	2.23	2.37	2.48	2.55	2.65	2.93	3.26	3.61	4.53	.....	.....
5..	.....	.....	.....	0.01	0.20	0.83	1.48	1.78	1.81	1.86	1.99	2.19	2.36	2.45	2.52	2.68	2.95	3.28	4.04	4.96	.....
6..	.....	.....	.....	0.00	0.03	0.38	1.07	1.61	1.79	1.81	1.85	1.96	2.16	2.31	2.43	2.54	2.70	2.98	3.57	4.46	.....
7..	.....	.....	.....	.....	0.00	0.09	0.61	1.29	1.72	1.80	1.80	1.83	1.93	2.10	2.29	2.46	2.56	2.73	3.25	3.98	.....
8..	.....	.....	.....	.....	.....	0.01	0.22	0.84	1.47	1.77	1.80	1.80	1.82	1.91	2.07	2.37	2.48	2.58	3.03	3.53	.....
9..	.....	.....	.....	.....	.....	0.00	0.03	0.39	1.05	1.60	1.79	1.80	1.80	1.81	1.90	2.23	2.42	2.49	2.78	3.24	.....
10..	.....	.....	.....	.....	.....	.....	0.00	0.10	0.58	1.25	1.70	1.80	1.80	1.80	1.82	2.00	2.32	2.44	2.63	3.03	4.78
11..	.....	.....	.....	.....	.....	.....	.....	0.00	0.20	0.80	1.45	1.76	1.80	1.80	1.80	1.85	2.14	2.41	2.54	2.73	4.28
12..	.....	.....	.....	.....	.....	.....	.....	.....	0.03	0.35	1.01	1.57	1.78	1.80	1.80	1.81	1.94	2.29	2.48	2.60	3.82
13..	.....	.....	.....	.....	.....	.....	.....	.....	0.00	0.08	0.54	1.22	1.67	1.80	1.80	1.80	1.85	2.06	2.43	2.56	3.40
14..	.....	.....	.....	.....	.....	.....	.....	.....	.....	0.00	0.17	0.75	1.38	1.74	1.80	1.80	1.81	1.90	2.39	2.52	3.18
15..	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0.02	0.31	0.93	1.53	1.78	1.80	1.80	1.82	2.33	2.45	3.06
16..	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0.00	0.06	0.50	1.15	1.65	1.80	1.80	1.80	2.14	2.42	2.92
17..	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0.00	0.17	0.68	1.33	1.80	1.80	1.80	1.94	2.40	2.70
18..	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0.02	0.27	0.88	1.76	1.80	1.80	1.84	2.34	2.58
19..	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0.00	0.05	0.41	1.61	1.80	1.80	1.80	2.16	2.54
−20..	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0.00	0.13	1.20	1.80	1.80	1.80	1.97	2.52

The data of Table V are plotted in Figure 1. At low temperatures the ionization of the metals is practically complete, with falling pressure, before that of the hydrogen begins, so that the curves run horizontally for some distance; at high temperatures the two overlap, and the curves are smoother. Similar changes of slope in the lower part of the figure show that, at the lower temperature, the

TABLE VII

MEAN IONIZATION AS A FUNCTION OF T AND $(1-\beta)/\beta$; VALUES OF LOG $(1/X)$

T	LOG q	$(1-\beta)/\beta$				
		1	0.1	0.01	0.001	0.0001
25200°......	−3.00	0.00	0.00	0.00	0.00	0.08
16800......	3.71	0.00	0.00	0.00	0.09	0.36
12600......	4.21	0.00	0.00	0.13	0.44	0.90
10080......	4.60	0.02	0.16	0.53	1.02	1.43
8400......	4.91	0.23	0.64	1.11	1.50	1.76
7200......	5.18	0.75	1.22	1.57	1.77	1.88
6300......	5.41	1.33	1.64	1.77	1.85	2.00
5600......	5.62	1.70	1.79	1.82	1.94	2.17
5040......	5.80	1.80	1.82	1.90	2.08	2.32
4582......	5.96	1.80	1.86	2.04	2.28	2.46
4200......	6.12	1.84	1.97	2.20	2.40	2.56
3877......	6.26	1.91	2.12	2.33	2.48	2.63
3600......	6.38	2.07	2.29	2.43	2.55	2.74
3360......	6.51	2.21	2.39	2.48	2.62	2.85
3150......	6.62	2.35	2.46	2.56	2.73	3.01
2800......	6.82	2.47	2.56	2.72	2.98	3.30
2520......	7.00	2.56	2.70	2.95	3.26	3.60
2291......	7.17	2.70	2.93	3.23	3.54	4.00
1939......	7.46	3.15	3.40	3.80	4.30	4.80
1680......	−7.71	3.64	4.12	4.61	....	....

ionizations of the alkali metals and of Mg and Fe are rather well separated. At the lowest temperatures Na and K are also separated.

4. It is often desirable to know the degree of ionization X as a function of the whole gas pressure p. The values of log p corresponding to the values of Table V were calculated by

$$p=(1+X)\frac{P_e}{X} \tag{5}$$

and plotted (Fig. 2) against those of log $(1/X)$. The curves are of much the same shape as before but run more obliquely.

From a large-scale diagram were read the values given in Table VI, which is arranged like the other tables. The values have been checked by duplicate graphical determination. The last figure may be in error by one or occasionally two units, but this is of no practical importance.

5. The range covered by the tables greatly exceeds that likely to be met in stellar atmospheres; for in these, radiation pressure is probably always a sensible fraction of the gas pressure. For the radiation pressure, in atmospheres,

$$q=2.515\times 10^{-21}T^4 .$$

By setting as usual $q/p=(1-\beta)/\beta$, the gas pressure corresponding to any given value of this ratio and the temperature follows, and the ionization can be found from Table VI. The results are given in Table VII.

For a star of the sun's mass and composed mainly of hydrogen, $(1-\beta)/\beta$ should be of the order of 0.01. The ratio 0.1 corresponds to a giant, 0.001 to a dwarf. The difference in temperature between giants and dwarfs having the same atmospheric ionization is conspicuous in the table.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
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THE SPECTROSCOPIC BINARY B.D. $-18^\circ 3295$

By RALPH N. VAN ARNAM

ABSTRACT

The *orbit* of B.D. $-18^\circ 3295$ (Boss 3138) was revised with the aid of *14 new spectrograms*. The *new period* is 2.96310 days; $\gamma = +3$ km/sec.; $e = 0.057$; $\omega = 68.^\circ 8$; $T = 1931$ Feb. 5, $21^h.239$ U.T.; $K_1 = 120.5$ km/sec.; $K_2 = 225$ km/sec.; $a_1 \sin i = 4{,}900{,}000$ km; $a_2 \sin i = 9{,}150{,}000$ km. The secondary spectrum was measured for the first time, and a *mass ratio* of secondary to primary of 0.53 determined. The minimum masses of the primary and secondary components are 8.2 and $4.4\odot$, respectively.

The *contours of the helium line* λ *4472* indicate a radial equatorial velocity of rotation of the order of 100 km/sec. From the contour areas of the two components of λ 4472 the *ratio of the radii* of the stars is found to be 1.5.

The variation in the radial velocity of B.D. $-18^\circ 3295$ ($\alpha\, 11^h 55^m.8$, $\delta - 19^\circ 6'$, spectral type B3, photographic mag. 5.3)[1] was announced by W. S. Adams in 1912.[2] The star's spectrum is characterized by faint and rather diffuse lines. In addition to the four Mount Wilson observations of 1911, three plates[3] were secured at the Yerkes Observatory in 1915, one in 1918; and thirty-one were obtained at Ottawa in the years 1916 and 1917. From the thirty-one Dominion Observatory plates J. B. Cannon derived orbital elements using a period of 1.50307 days.[4] This orbit is of particular interest because it is one of several in which the published period is erroneous, as was shown by Otto Struve in 1928.[5]

Because of its southern declination, B.D. $-18^\circ 3295$ is rather difficult to observe in northern latitudes. All plates must be taken when the star is near the meridian, and the observations are consequently grouped at certain epochs in the period of velocity variation.

In general, there are three periods which give plausible velocity-curves for these ambiguous cases. According to Hagen,[6] the follow-

[1] This star has been designated as 31 Crateris in accordance with Flamsteed's catalogue. It has also been referred to as 31 Corvi because of its position in Corvus in agreement with the zoning of the International Astronomical Union and with other star charts. Since the latter designation violates Flamsteed's numbering for Corvus, it seems best that the star be known as B.D. $-18^\circ 3295$ or Boss 3138.

[2] *Astrophysical Journal*, **35**, 176, 1912. [3] *Ibid.*, **64**, 25, 1926.

[4] *Publications of the Dominion Observatory, Ottawa*, **4**, 125, 1917.

[5] *Popular Astronomy*, **36**, 411, 1928.

[6] *Die veränderlichen Sterne*, **1**, 624, 1920.

ing simple relations exist among the three periods if the inequalities $p_1 > p_2 > 1^d > p_3$ are assumed:

$$p_2 = \frac{p_1}{p_1 - 1}\,; \qquad p_3 = \frac{p_2}{2p_2 - 1}\,; \qquad p_1 = \frac{p_3}{1 - p_3}\,.$$

For observers in northern latitudes working on the orbits of southern stars it is desirable to combine the observations made at observatories differing widely in longitude, if such observations are

TABLE I

RADIAL VELOCITIES OF B.D. $-18°3295$

Date U.T.	Obs.	Phase	Vel. I	Vel. II
1931 Feb. 5.382	σ, S	0.782	+131	−178
8.397	σ, S	0.837	+130	....
21.382	Mgn, S	1.965	−107	....
Mar. 11.316	σ, Sch, S	2.126	−109	+234
17.327	σ, Sch, S	2.210	−117	+216
31.218	Mgn, S	1.284	+ 43	....
Apr. 2.257	Sch, S	0.361	+ 91	....
6.236	Mgn, S	1.378	+ 26	....
11.237	Mgn, S	0.452	+108	....
May 3.211	Sch, S	1.680	− 39	+216
4.093	σ, S	2.560	− 83	+133
4.187	σ, S	2.660	− 59	....
14.147	Sch, S	0.767	+118	....
26.148	Sch, S	0.915	+113	....

The names of the observers are designated as follows: σ = O. Struve; Mgn = W. W. Morgan; Sch = H. Schwede; S = F. R. Sullivan.

available, in order that the gaps in the velocity-curve may be filled. If the three different periods are applied, the true value will be presumably the one in which the observations exhibit the least scattering from the velocity-curve.

I have lately reinvestigated the question of the true period of B.D. $-18°3295$ in the light of some new observational evidence. Fourteen spectrograms obtained at the Yerkes Observatory during the winter and spring of 1931 were measured (see Table I) and discussed together with all the available observations in an attempt to settle definitely which choice of period gives the best representation. It appears that a slight modification of the longer period, p_1, satisfies the measures best. This is in harmony with Struve's result which was obtained from the Ottawa observations alone.

Using Cannon's period, $p_2 = 1.50307$ mean solar days $= 1.50719$ sidereal days, we find from the foregoing relations $p_1 = 2.97165$ sidereal days $= 2.96354$ mean solar days, this being the most likely value of the period. The alternative period is $p_3 = 0.74822$ sidereal days $= 0.74618$ mean solar days.

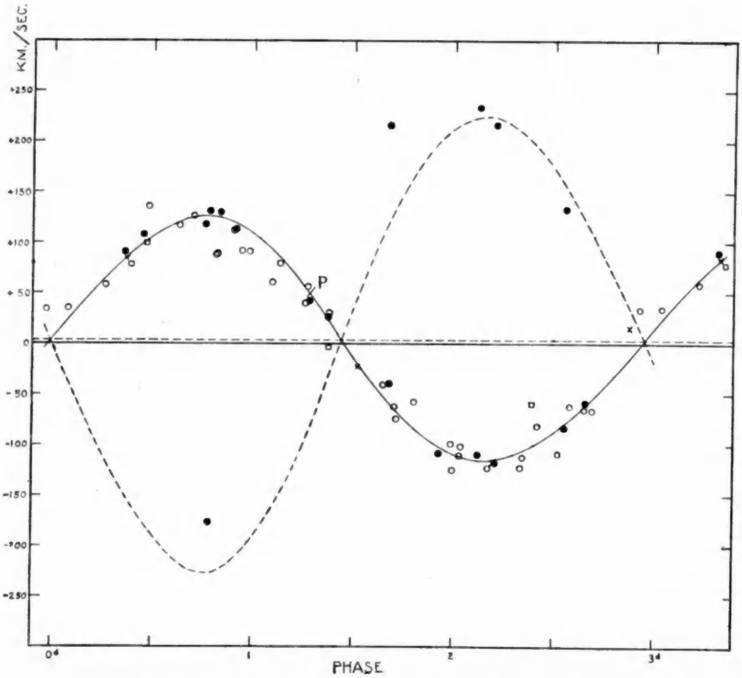

FIG. 1.—Radial velocity curve of B.D. $-18°3295$. Period 2^d96310. Phase zero corresponds to 1931 Feb. 4, 14^h40 U.T. Observations: o, Ottawa; •, 1931 Yerkes; □, old Yerkes; x, Mt. Wilson.

By trial I found that the period 2.96310 days gives the most satisfactory fit to all the observations. This velocity-curve is shown in Figure 1. The largest residual, of approximately 40 km/sec., is given by a Yerkes plate taken on March 23, 1915; the largest residual published by Cannon is -74 km/sec.; and the largest residual for the 1931 measures is about 20 km/sec.

Figure 2 represents the distribution of the observations, using a small but necessary modification of Cannon's period, viz., 1.5032 days. It seems unlikely that this value can be improved in view of

the scattering of the points. It is interesting that the four Mount Wilson observations, represented by crosses, are displaced with respect to the curve. This effect is conceivably due to the difference in longitude between Ottawa and the Pacific Coast. The curve itself has no particular significance, being merely a free-hand attempt to satisfy the observations.

The third period also gave more scattering of the observations than the adopted value.

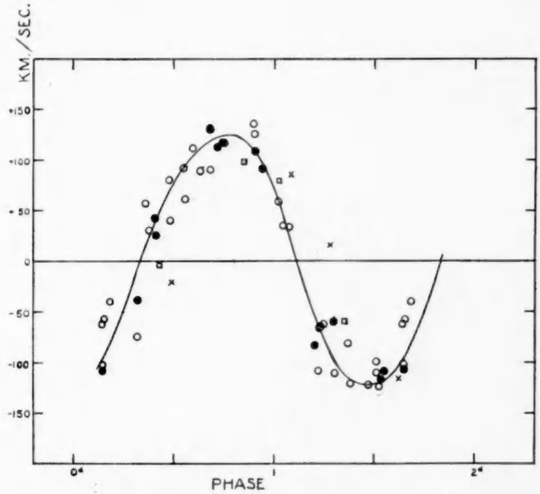

FIG. 2.—Velocity curve of B.D. $-18°3295$. Period 1^d5032. Phase zero corresponds to 1931 Feb. 4, 16^h80 U.T.

I was able to measure a few lines in the spectrum of the secondary component on five plates at epochs near the extremes of the velocity-curve. These lines are exceedingly faint. It is rather interesting that most of these points occur at secondary maximum rather than being more evenly distributed at maximum and minimum. While the residuals are large for two of the observations, the semi-amplitude of the secondary curve appears to be fairly well determined. The mass ratio of the secondary to the primary is found to be 0.53.

The curve in Figure 1 represents elliptic motion as calculated from the orbital elements given in Table II. The elements were derived from a free-hand curve by the method of Lehmann-Filhés. Correc-

tions indicated by an ephemeris were so small that the initial set of elements was adopted as final.

TABLE II

ORBITAL ELEMENTS OF B.D. $-18°3295$

Element		
Period	P	$2^{d}96310$
Velocity of system	γ	$+3$ km/sec.
Eccentricity	e	0.057
Longitude of apse	ω	$68°8$
Time of periastron	T	1931 Feb. 5, $21^{h}239$ U.T.
Semi-amplitude I	K_1	120.5 km/sec.
Semi-amplitude II	K_2	225 km/sec.
Semi-axis major I	$a_1 \sin i$	4,900,000 km
Semi-axis major II	$a_2 \sin i$	9,150,000 km
Minimum mass I	$m_1 \sin^3 i$	$8.2\odot$
Minimum mass II	$m_2 \sin^3 i$	$4.4\odot$
Mass ratio	m_2/m_1	0.53

Because of the combination of observations from different observatories, their distribution over a twenty-year period of time, and the poor character of the star's spectrum, a least-squares adjustment was not deemed necessary.

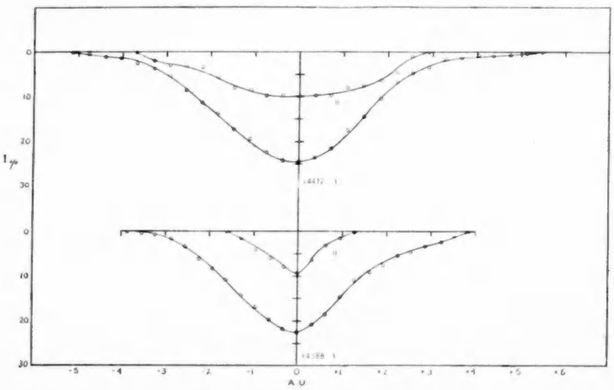

FIG. 3.—Observed contours of helium lines in B.D. $-18°3295$. I = brighter component; II = fainter component.

A study was made of the contours of the helium lines λ 4388 and λ 4472 with a view to determining the rotation and relative dimensions in this binary system. The observed contours are shown in Figure 3. Those of the primary component are based on the means

of measures from six microphotometric tracings. The contours of the secondary component are the means of measures on two tracings.

We shall assume that the two components of this binary are similar in spectral type and that the periods of revolution and of rotation are equal. Struve[1] has shown that "the observed areas of the contours of the two components are proportional to the intensities of their continuous spectra." Consequently,

$$M_1 - M_2 = -2.5 \log \frac{A_1}{A_2} ,$$

where the M's are the absolute magnitudes, and the A's are the areas of the observed contours. Furthermore,

$$\log \frac{r_1}{r_2} = 0.2(M_2 - M_1) .$$

From the areas of the observed contours of λ 4472 we find $M_1 - M_2 = 0.95$, and the ratio of the radii of the two stars is 1.5.

An approximate quantitative determination of the rotational velocity of the primary star was obtained from the contour of λ 4472. Theoretical contours[2] calculated from the line Mg^+ 4481 in the spectrum of Sirius, for several assumed Doppler displacements of the limb of the star,[3] were compared with the observed contour of λ 4472. This procedure gave a radial component of the rotational velocity of the order of 100 km/sec.

I wish to express my appreciation to Dr. Otto Struve and to Dr. C. T. Elvey for their interest in this work and suggestions offered. I am indebted to Director E. B. Frost for the privilege of spending the summer of 1931 at the Yerkes Observatory.

YERKES OBSERVATORY
August 1931

[1] *Astrophysical Journal*, **72**, 12, 1930.

[2] *Monthly Notices of the Royal Astronomical Society*, **89**, 226, 1929.

[3] C. T. Elvey, *Astrophysical Journal*, **71**, 222, 1930

PHOTO-ELECTRIC COLORS OF STARS OF EARLY TYPE IN CEPHEUS

By C. T. ELVEY and T. G. MEHLIN

ABSTRACT

The colors of 49 stars of spectral types B3 and earlier in the region of Cepheus have been determined with the photo-electric photometer attached to the 40-inch telescope of the Yerkes Observatory. Stars as faint as the ninth magnitude were observed.

The color excesses of the stars fail to correlate with the intensities of the interstellar calcium line, and hence the material producing the selective scattering of the star's light is not coexistent with the calcium atoms.

The correlation of color excess with apparent magnitude gives a coefficient of +0.49. This correlation may be the effect of the scattering of light produced by the partially obscuring dark clouds observed on photographs of the region. It is found that the stars of small color excess are in the unobscured regions and those of large color excess are in the obscured regions.

The cause of the reddening of a star's light has been discussed by many. R. J. Trumpler[1] and others have attributed it to the scattering of light by material in interstellar space. On the other hand, B. P. Gerasimovič[2] and others have attributed it to conditions originating in the star itself. Recently, C. T. Elvey[3] has shown that there are two factors entering into the question, namely, the scattering of light by interstellar material, which statistically may be considered as a phenomenon of distance, and the total absorption of the hydrogen lines. Otto Struve[4] has shown that for some of the brighter stars there is a relationship between the color of the star and the intensity of the interstellar calcium line in the star's spectrum. He has suggested to us that we make a detailed study of this relation in a region of the sky for which the interstellar absorption lines are abnormally strong. We have taken as our program all stars of spectral classes B3 and earlier of the *Henry Draper Catalogue* in the region of Cepheus from 21^h to 22^h30^m and between declinations $+40°$ and $+70°$. This includes the region for which Struve[5] has made a special study of the intensities of the calcium line.

[1] *Bulletin of the Lick Observatory*, **14**, 154, 1930; *Publications of the Astronomical Society of the Pacific*, **42**, 214, 267, 1930.

[2] *Circular of the Harvard College Observatory*, No. 339, 1929.

[3] *Astrophysical Journal*, **74**, 298, 1931.

[4] *Observatory*, **52**, 52, 1929; *Die Naturwissenschaften*, **17**, 717, 1929.

[5] *Astrophysical Journal*, **65**, 63, 1927.

The colors were determined with a photo-electric photometer attached to the 40-inch telescope of the Yerkes Observatory. The instrument has been described by Joel Stebbins.[1] The method of reduction has been somewhat modified, since with this pair of filters (Y_1B_1) the corrections for extinction were not the same as in the previous work.[2] The factor $(f_y - f_b)$ which modifies the extinction was determined on one of the nights of good transparency from several observations of the standard star Alpha Cephei. The color determined on that night was used with any other observation of another night to obtain the factor to reduce the observations of other stars. All stars were observed on at least two nights.

The transmissions of the filters Y_1B_1 were roughly determined by photographing on a calibrated plate the sky spectrum with and without the filter in front of the slit of the Bruce spectrograph. The transmissions when combined with the sensitivity of a sensitized potassium cell and the approximate transmission of the 40-inch objective gave the effective wave-lengths of λ 4750 for Y_1 and λ 4250 for B_1. There is an overlapping for the two filters by the amount of the distance between their effective wave-lengths. The scale of colors for Y_1B_1 is 0.34 times that of the pair of filters, Y_2B_2, used in the former investigation.

A theoretical curve has been computed for the variation of the colors with spectral class by using the effective wave-lengths determined for the combination of the cell and the pair of filters, and by assuming that the stars radiate as black-bodies with the temperatures for the various spectral classes as given by Russell, Dugan, and Stewart.[3]

This curve is shown in Figure 1. A group of bright stars which have small color excesses in the former investigation were observed and used for calibration. They are listed at the end of Table I and are shown in Figure 1 by the open circles. All the computed colors have been changed by a constant amount to make the stars used for calibration agree with the theoretical curve. This is justified since the filters were chosen so as to give about equal response under the conditions of observing for a star of early type. The star in spectral

[1] *Ibid.*, **74**, 289, 1931.

[2] Elvey, *loc. cit.*

[3] *Astronomy*, **2**, 753, 1926.

class A5 shown as a dot is 5 *α* Cephei, the standard star to which all observations have been referred.

The observed colors of the stars of class B3 and earlier in the region of Cepheus are listed in Table I. Since in most cases there are only two observations of a given star, the average deviation is merely half of the difference of the colors of the star as observed

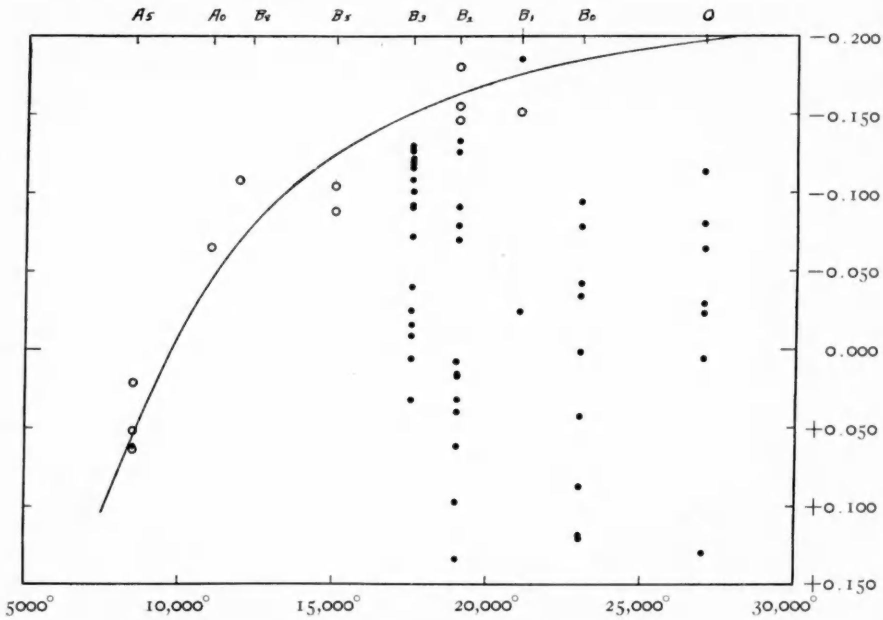

FIG. 1.—Colors of stars in the region of Cepheus. The circles represent bright stars used for calibration. The curve gives the theoretical colors for "black bodies" for the given temperatures and the filter combinations.

on the two nights. Any systematic difference between a group of stars observed on two nights is small in comparison to the errors of observation. The errors of observation include variations in the transparency of the sky from star to star. The mean of the average deviations of each set of observations is ± 0.0086 mag. and the largest deviation is ± 0.032 mag. In column 5 of the table are listed the color excesses, that is, the observed minus the theoretical color. An examination of the table, or Figure 1, shows that most of the color excesses are positive, that is, the star is redder than a black-

TABLE I

H.D.	m_v	Spec.	Color	E	Ca^+ Int.	Remarks
200857	7.16	B2	+0.134±004	+0.296	4.8	
202214	5.65	B2	− .079±007	.083	4.6	
202253	7.70	B3	− .016±010	.134	3.0	
203025	6.41	B3	− .009±013	.141	7.0	
203064	5.06	Oe5	− .113±000	.084	4.4	
203374	6.64	Bop	+ .002±004	.187	5.8	
203467	5.18	B3p	− .130±008	.020	2.7	6 Cephei
203938	7.10	B2	+ .098±010	.260	4.2	
204116	7.94	B0	+ .119±027	.304	2.8	
204150	7.61	B3	− .092±010	.058	3.8	
204722	7.52	B2	− .070±002	+ .092	1.0	
205021	3.32	B1	− .185±022	− .010	0.5	8 β Cephei
205139	5.52	B0	− .042±013	+ .143	4.1	
205196	7.36	B0	+ .121±002	.306	3.7	
206165	4.87	B2p	+ .016±007	.178	7.0	9 Cephei
206267	5.64	Oe5	− .029±010	.168	5.5	
206327	8.5	Oe5	− .023±007	.174	3.3	
206672	4.78	B3	− .127±001	.023	4.2	80 π^1 Cygni
206773	6.98	Bop	− .034±012	.151	4.4	
207198	5.97	B2	+ .017±000	.179	7.6	
207308	7.9	B2	+ .008±008	.170	5.7	
207329	7.45	B2	+ .032±013	.194	1.6	
207330	4.26	B3	− .128±008	.022	5.0	
207538	7.03	B3	+ .032±020	.182	2.4	
207793	6.56	B2	+ .062±003	.224	5.5	
208095	5.54	B3	− .120±008	.030	1.4	
208106	8.1	B3	− .025±004	.125	5.0	
208185	7.7	B3	− .040±008	.110	8.2	
208218	6.76	B1	− .024±030	.151	4.8	
208220	9.0	Oe5	− .064±019	.133	3.0	
208392	7.10	B3	+ .006±002	.156	10.0	
208682	5.85	B2	− .133±003	.029	4.1	
208904	7.60	B3	− .101±009	.049	2.8	
208905	6.90	B3	− .072±006	.078	4.8	
208947	6.28	B3	− .116±004	.034	5.6	
209339	6.48	B0	− .078±000	.107	5.5	
209481	5.50	B0	− .094±007	.091	4.1	14 Cephei
209961	6.16	B3	− .122±008	.028	4.5	
209975	5.17	Oe5	− .080±004	.117	6.5	19 Cephei
210809	7.7	B2	− .091±009	.071	5.8	
210839	5.19	Od	+ .006±004	.203	6.1	22 λ Cephei
211853	9.0	Ob	+ .130±032	.327	....	
212044	7.08	B2p	− .126±002	.036	5.5	
212222	6.27	B3	− .108±002	.042	3.5	
212455	8.4	B2	+ .040±004	.202	9.0	
213087	5.66	B0	+ .043±018	.228	4.7	
213322	6.59	B3	− .091±006	.059	3.0	
213405	8.4	B0	+ .088±004	.273	3.5	
213420	4.54	B3	−0.118±006	+0.032	3.6	
886	2.87	B2	−0.146	....	....	88 γ Pegasi
3360	3.72	B3	− .155	....	....	17 ζ Cassiopiae
4636	5.03	B9	− .108	....	....	25 ν Cassiopiae
8538	2.80	A5	+0.052	....	....	37 δ Cassiopiae

TABLE I—*Continued*

H.D.	m_v	Spec.	Color	E	Ca^+ Int.	Remarks
11636	2.72	A5	+0.064			6 β Arietis
13161	3.08	A5	+ .021			4 β Trianguli
16582	4.04	B2	− .180			82 δ Ceti
22928	3.10	B5	− .088			39 δ Persei
23630	2.96	B5p	− .104			25 η Tauri
24760	2.96	B1	− .151			45 ε Persei
203280	2.60	A5	+ .062			5 α Cephei
218045	2.57	A0	−0.065			54 α Pegasi

body of the same assumed temperature. This is very probably a real effect, for, as was shown in the previous paper on the colors of the brighter stars of early types, the theoretical curve computed on the assumption that the stars radiate as black-bodies formed an envelope within the errors of observation about the bluest stars of each class.

Since the colors of the stars in this region have been adjusted to a system comparable with that of the former paper by means of the calibration stars, we may then indicate a few points of difference between the two groups of stars. The average color of the stars of type O in Cepheus is about class A1 as compared to class B8 for the bright stars (bright stars refers to the former paper). The B0 stars are of the color of A3 as against about B7, the B2 stars are of the color of A2 compared with B6, and the stars of B3 are of about B8, the same color as those of the same class among the brighter stars. In general, the stars of types earlier than B3 are redder in the region of Cepheus than for the brighter stars of the same classes.

In column 6 of the table are the intensities of the interstellar calcium line K for each of the stars. We are indebted to Professor Struve for these data, which have not been published individually. To test for any relationship between the intensity of the calcium line and the color excess of the star the values of one were plotted against those of the other, as is shown in Figure 2. It is immediately evident that there is no correlation. This is somewhat contrary to what has been expected, for Struve[1] has shown that the intensity of the interstellar calcium line is correlated with the distance of the star, and also that the colors are likewise related if we can attribute

[1] *Astrophysical Journal*, **67**, 353, 1928; *Monthly Notices of the Royal Astronomical Society*, **89**, 567, 1929.

the correlation with apparent magnitude as one of distance. This lack of correlation is, however, not a proof that the color of a star is not a function of its distance. However, it does show definitely that the major portion of the reddening of the light of the stars in the region of Cepheus which is due to material in interstellar space is not caused by the calcium atoms or any material coexisting with them.[1]

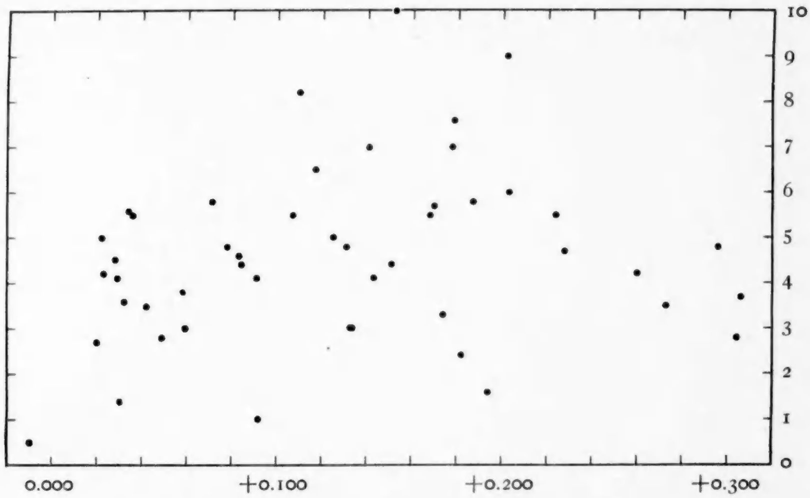

FIG. 2.—A plot of the color excesses (abscissae) of stars in the region of Cepheus against the intensities of the interstellar calcium line K.

Next the relationship between color and apparent magnitude was investigated. Figure 3 shows a plot of the color excess of a star against its apparent magnitude. There is a large scatter in the points in the diagram but there is a tendency for the stars of fainter apparent magnitude to be associated with the larger positive color excesses. The coefficient of correlation was computed and found to be +0.49. The "regression lines" are shown in the figure. The full line represents the averaging in the color excesses, and the dotted

[1] It may be that we have here a superposition of two effects, i.e., a reddening which is correlated with the intensity of the interstellar calcium line and one which is caused by the partially obscuring cloud noted below. Struve found (unpublished) that there was a weak correlation between his calcium intensities and the color excesses of the stars in Elvey's paper.

line the averaging in the apparent magnitudes. If we consider the apparent magnitude as a measure of the distance of the star, then we must see what is the effect of a variation in the absolute magnitudes of the stars. We have placed together all stars of spectral types B3 and earlier, and hence are including the variation of absolute magnitude within any one spectral subdivision as well as the change with spectral type. The change with spectral type is about 2 mags., according to W. S. Adams and A. H. Joy,[1] and the probable spread within a spectral subdivision for a small group of stars would

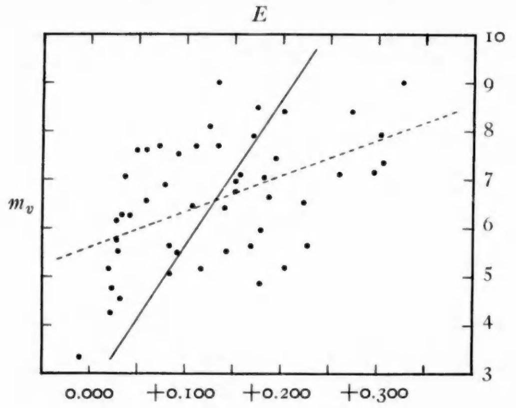

FIG. 3.—Correlation of color excess and apparent magnitude for the class B stars in Cepheus. The "regression lines" are shown and the coefficient of correlation is 0.49.

be about 2 mags. This would mean that some of the fainter stars would be decidedly too blue for their apparent magnitudes. On the other hand, there is an absolute magnitude effect resulting from the total absorption of the hydrogen lines, that is, the dwarf stars have widened hydrogen lines caused by the Stark effect and this produces a reddening of the star's light. The two effects due to the absolute magnitudes of the stars tend to cancel each other. Since there is considerable scattering of the observations in Figure 3, it would seem to indicate that some other effect was present. From the agreement of the observations it is not instrumental but is no doubt in the color of the star's light as we receive it. The most natural thing to look for is the non-uniformity of the distribution of the material

[1] *Astrophysical Journal*, **57**, 294, 1923.

in interstellar space which is producing the selective scattering of the starlight. A glance at any photograph of a section of the Milky Way shows patches of bright and dark nebulosity.

Plate XVI is a photograph of our region taken by Professor F. E. Ross with the 2-inch Zeiss camera at Mount Wilson. On the page opposite from the plate is a chart of the same region showing by the size of the circles the color excesses of the stars observed. The circles in order of size represent color excesses less than 0.075, 0.076–0.150, 0.151–0.225, and greater than 0.225 mags., respectively. In the chart are marked a few of the regions of obscuration which will help to locate the stars. It is readily noted that all the stars showing the largest color excesses are located in regions of obscuration and that those of smallest color excesses are in the unobscured regions, excepting for one bright star which no doubt belongs to the foreground. An examination of the photograph shows regions which seem to be areas of only partial obscuration, principally those marked on the chart, which are probably in front of the Cepheus cloud. The smaller very dark areas are probably more distant but in many cases still in front of the star cloud. The colors do not seem to be related to these smaller dark areas but to the more general areas of smaller opacity. It will even be noted that the color excesses of medium values are associated with regions that are borderline cases, taking, for example, the group of stars to the east and the south of 5 α Cephei. It seems quite definite that most of the color excesses are due to clouds of obscuring material between the stars and the observers.[1] The correlation noted above, color excess with apparent magnitude, may be mainly this effect since the greater the obscuration the fainter the star and the redder it will become.

The lack of correlation with the intensity of the interstellar calcium lines is again exemplified, for in Struve's work he found no correlation of the intensities with the obscuring clouds and the colors show a distinct relationship.

We may conclude that in the region of Cepheus the largest factor producing a reddening of the star's light is the extensive clouds of

[1] Frederick H. Seares and Edwin Hubble, *ibid.*, **52**, 8, 1920, and Hubble, *ibid.*, **56**, 400, 1922, have shown that stars imbedded in nebulosity have large positive color excesses.

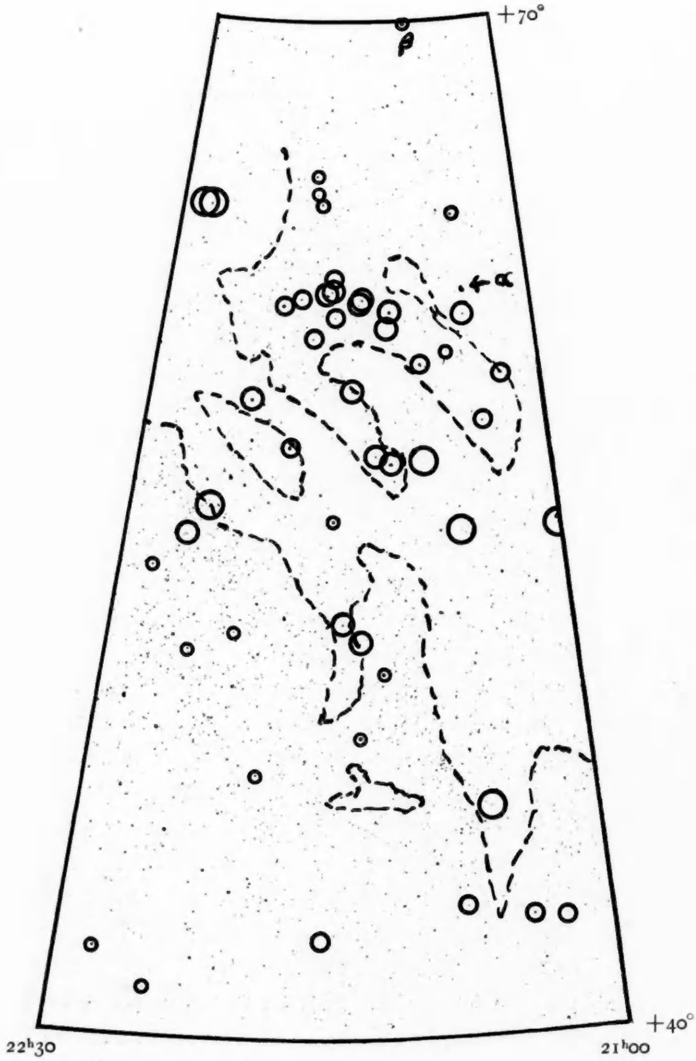

FIG. 4.—Chart of the region in Cepheus showing the color excesses of the stars by the size of the circle. The stars of smallest color excess (small circles) are found mostly in the unobscured areas.

PLATE XVI

Photograph of the Region in Cepheus Taken by F. E. Ross with the 2-Inch Zeiss Camera at Mount Wilson

material which is producing a partial obscuration of the stars behind it. Also, that this material is not coexistent with the calcium atoms producing the line absorption.

We are very much indebted to Professor Ross for permission to use his photograph, to Professor Struve for the unpublished intensities of the interstellar calcium line, and to Miss Christine Westgate who assisted with the reductions of the observations.

YERKES OBSERVATORY
December 1931

VARIATIONS OF RADIAL VELOCITY AND OF INTENSITY OF SPECTRAL LINES IN $12\alpha^2$ CANUM VENATICORUM

By GOLDENA FARNSWORTH

ABSTRACT

Velocity curves of α^2 Canum Venaticorum were determined for several individual lines and compared with intensity curves determined from the same plates. A *revised period* depending upon observations of Belopolsky and upon spectrograms obtained in 1931 at the Yerkes Observatory was used. It is suggested that the observed changes in radial velocity are apparent and are caused by the effect of blending with other lines, near minimum of intensity of the lines considered. *The star should probably be regarded as having a constant radial velocity* within the precision of the measurements.

Thirty-two plates taken with the single-prism spectrograph attached to the 40-inch refractor of the Yerkes Observatory were used in this study. All the spectrograms, listed in Table I, were made on Eastman Process plates and were, as a whole, of exceptional quality. The exposure time was approximately 30 minutes.

Visual estimates of the intensity of the *Eu* II lines $\lambda\lambda$ 4205 and 4130 and the *Fe* II line λ 4233 were obtained from each plate. Assuming the period 5.469 days and taking January 13, 1931, 7:49 U.T. as zero phase, curves were plotted showing the variation of intensity with phase. Using Belopolsky's[1] data on the spectral line λ 4130, a similar curve was drawn. When the period was computed from the foregoing curves, the value 5.4695 days was obtained. This is the period that was used in this work.

Using two plates that were taken near the maximum and minimum phase of the spectral line λ 4205, a group of sixteen strong lines was chosen from the spectrum for special study. These included lines of group I like λ 4205, of group II like λ 4233, and lines approximately constant in intensity like λ 4549.5. The radial velocities of these lines were measured on all of the plates, and graphs showing radial velocity as a function of phase were made for each line. A similar set of graphs was made using the estimates of intensity and phase. These results are recorded in Figures 1 and 2. The circles on the radial velocity graphs represent independent

[1] *Astronomische Nachrichten*, **196**, 6, 1913–1914.

measurements on four of the plates by O. Struve. The wave-lengths used are recorded in I.A. The visual estimates of the intensities were made on an arbitrary scale of ten—i.e., ten denotes a very strong line and zero absence of the line.

TABLE I

RADIAL VELOCITY OF α^2 CANUM VENATICORUM

Plate No.	Date	(U.T.)	Phase	Mean Radial Velocity*	Quality†	Observer‡
1R 9593	1931 Jan. 14	7^h49^m	1^d000	−1.7	fg	Sch, σ, S
1R 9605	17	8:46	4.039	−4.6	vp	M, S
1R 9611	20	7:14	1.506	−1.9	g	Sch, M, S
1R 9617	25	7:49	1.062	−4.0	faint	Sch, M, S
1R 9620	26	8:16	2.080	−5.3	dense	Sch, M, S
1R 9622	30	7:16	0.580	−0.6	fg	M, σ, S
1R 9632	Feb. 5	7:48	1.123	+1.1	dense	σ, S
1R 9636	8	8:18	4.144	−3.1	fg	Sch, σ, S
1R 9652	21	5:49	0.633	+1.4	g	Sch, S
1R 9663	25	6:17	4.653	+3.1	p	Sch, S
1R 9676	Mar. 23	6:07	3.301	+2.4	faint	σ, S
1R 9681	31	7:15	0.410	+6.0	dense, fg	M, σ, S
1R 9686	Apr. 2	4:35	2.299	+1.7	dense	Sch, S
1R 9691	5	8:26	5.459	−1.8	fg	M, S
1R 9695	6	2:58	0.763	+4.6	vg	M, S
1R 9701	8	4:54	2.843	−7.5	g	Sch, S
1R 9704	10	7:52	4.967	−2.8	fg	M, S
1R 9712	14	4:07	3.342	+0.7	g	Sch, S
1R 9732	30	5:16	2.982	−3.4	fg	M, S
1R 9748	May 17	3:49	3.515	−1.5	fg	σ, S
1R 9761	31	4:35	1.140	+1.7	g	M, S
1R 9767	June 1	6:53	2.236	−4.0	g	Sch, S
1R 9772	11	3:42	1.165	−0.9	g	Sch, S
1R 9776	15	2:10	5.101	+0.8	faint, fg	σ, K, S
1R 9777	15	2:45	5.126	−3.5	g	K, S
1R 9781	17	2:12	1.634	+2.4	vg	Sch, S
1R 9785	19	2:14	3.635	−0.9	v dense	Sch, S
1R 9787	20	2:08	4.631	+0.9	f	σ, V, S
1R 9789	25	2:52	4.192	+2.0	g	Sch, S
1R 9792	29	3:12	2.737	−1.2	f	Sch, S
1R 9795	30	2:20	3.701	−1.4	faint, fg	K, S
1R 9803	July 7	2:21	5.233	+1.4	vg	Sch, S

* Mean for λλ 4128, 4131, 4233, 4352, 4481, 4534, 4549.5, and 4584.

† g = good, f = fair, p = poor, v = very, fg = fairly good.

‡ K = P. C. Keenan, M = W. W. Morgan, Sch = H. F. Schwede, V = R. N. Van Arnam, S = F. R. Sullivan, σ = O. Struve.

In general, the results do not show complete correlation between radial velocity and intensity. In some cases the error of measurement obscures the result and no conclusion is possible. The best correlation is shown with the lines *Fe* II λ 4201 and *Cr* II λ 4558.6 of

group II and the constant line λ 4549.5. The lack of complete correlation in the case of the line λ 4584 is probably due to the fact that it is not easy to estimate the intensity of this line as it is very near

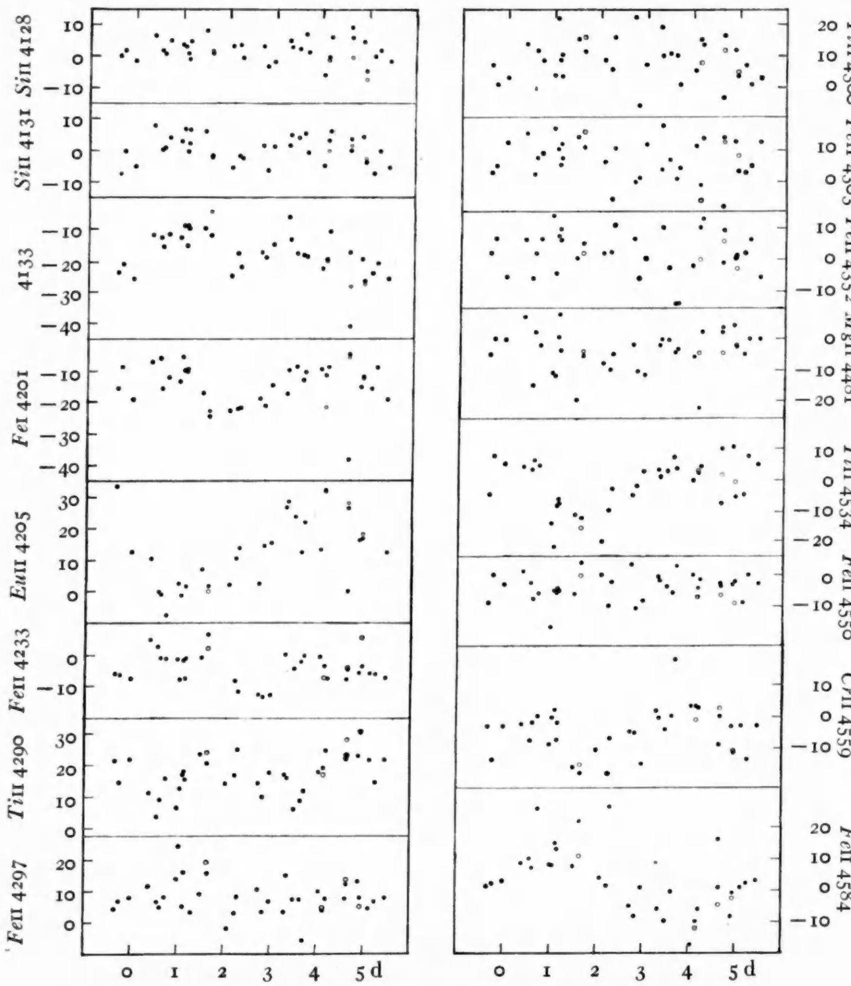

FIG. 1.—Radial velocities from individual spectral lines of 12α² Can. Ven.

another variable line. A later estimate of the intensity variation of this line gave a curve in which there was more definite correlation. The spectral lines λλ 4205 and 4534 have radial velocities and intensities definitely out of phase. The velocity curve for λ 4205 agrees

in phase with that published by Kiess[1] but is not (as he found) symmetrical with respect to the zero axis. During the progress of

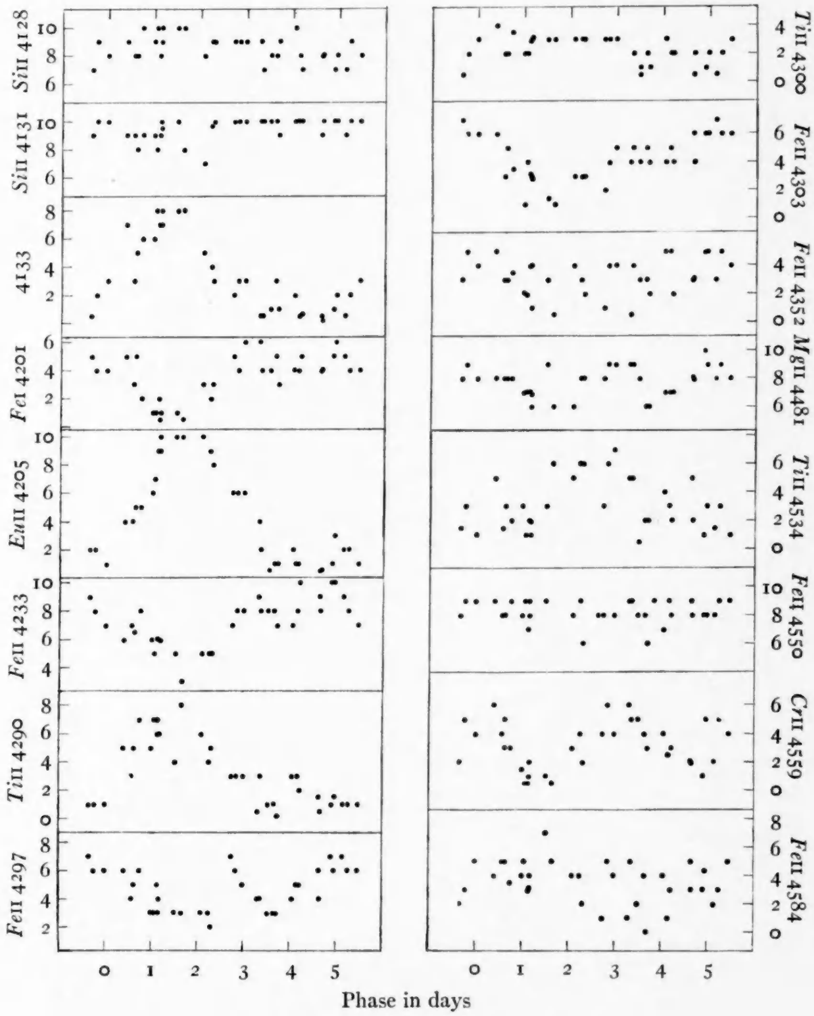

Fig. 2.—Intensities of spectral lines in 12α² Can. Ven.

the measurements on radial velocity it was suspected that the large displacement of the line λ 4205 at minimum phase was due to a

[1] *Publication of the University of Michigan Observatory*, 3, 106, 1923.

blend. In fact, there was some slight evidence on the plates that a double was being measured. This made it desirable to investigate further the question of blends.

The radial velocity of the star was determined by taking the mean of the results for lines λλ 4128, 4131, 4233, 4352, 4481, 4534, 4549.5, and 4584. These were the lines whose wave-lengths were most accurately known. The result was -0.6 km/sec. Using this for the velocity of the star, the wave-length of each of the lines measured was adjusted from its mean radial velocity and compared with laboratory and stellar wave-lengths taken from data on the star α Persei.[1] The results are recorded in Table II.

A study of this table gives rather convincing evidence that the apparent change in radial velocity of λ 4205 is due to the blend with *Mn* II at λ 4205.4. There is also evidence that the particular wave-lengths obtained for some of the other spectral lines of this star are due to blends. This, together with the fact that for some lines the changes in radial velocity are in phase with changes in intensity while for other lines the phases are reversed, would suggest that most or all of the observed variations in radial velocity are caused by blends. The star should probably be regarded as having a constant radial velocity.

An attempt was made to investigate the behavior of lines belonging to different stages of ionization. For this purpose several lines of *Fe* I (λλ 4045, 4143, 4198.5, 4325, 4383, and 4404) were selected for intensity estimates. Data were already available on the *Fe* II lines λλ 4233, 4296.5, 4303, 4352, 4584, and the *Fe* I line λ 4201 (Fig. 2). Unfortunately, most of the *Fe* I lines chosen proved to be too weak to give reliable data. The line λ 4325, which is fairly strong, seemed to belong to group II, as does the *Fe* I line λ 4201. There was slight evidence that some of the other neutral iron lines belonged to Group I. The *Fe* II lines λλ 4233, 4296.5, and 4303 seem to belong to group II; the *Fe* II lines λλ 4352 and 4549.5 (blend with *Ti* II) are approximately constant; and the *Fe* II line λ 4584 is probably a group I line. From the foregoing data no definite conclusion was

[1] The spectrum of α Persei by T. Dunham, Jr., *Contributions from the Princeton University Observatory*, No. 9, 1929.

TABLE II

WAVE-LENGTHS

λ Used for Radial Velocity Measurements (I.A.)	Mean Radial Velocity in Km/sec.	λ Corrected (I.A.)	λ (α Persei)	
			Stellar	Laboratory
4128.053	+ 2.25	4128.084	....	*Si* II 4128.053*
4130.884	+ 1.14	4130.900	....	*Si* II 4130.884*
4132.587	−17.03	4132.359	....	....
4201.004	−14.22	4200.813	4200.94	*Fe* I 4200.92
4205.046	+12.04	4205.224	4204.80	*Y* II 4204.69
			4205.05	*Eu* II 4205.04
				V II 4205.09
			4205.41	*Mn* II 4205.47
4233.264	− 4.82	4233.196	4233.19	*Fe* II 4233.16
				Cr II 4233.28†
			4233.58	*Fe* I 4233.61
4289.844	+16.54	4290.090	4289.46	*Ca* I 4289.36
			4289.80	*Cr* I 4289.73
				Ce II 4289.93†
			4290.26	*Ti* II 4290.22
4296.566	+ 7.88	4296.688	4296.60	*Fe* II 4296.56
				Ce II 4296.68†
				Ce II 4296.78†
4300.056	+ 8.92	4300.193	4300.05	*Ti* II 4300.05
			4300.35†	*Ce* II 4300.33
				Ti II 4300.57
4303.196	+ 7.38	4303.311	4303.20	*Fe* II 4303.18
			4303.67†	*Nd* II 4303.61
4351.840	+ 2.62	4351.878	4351.81	*Fe* II 4351.77
				Cr I 4351.77†
				Mg I 4351.94†
4481.230	− 4.21	4481.167	4481.25	*Mg* II 4481.13
				Mg II 4481.33
4533.972	− 1.75	4533.945	4534.02	*Ti* II 4533.97
				Fe II 4534.17†
4549.532	− 3.68	4549.476	4549.59	*Fe* II 4549.48
				Ti II 4549.62
4558.650	− 5.47	4558.576	4558.66	*La* II 4558.48†
				Cr II 4558.66
4583.847	+ 3.39	4583.899	4583.51	*Ti* II 4583.45
			4583.81	*Fe* II 4583.84

* Taken from Yerkes Observatory Reduction Tables. † Weak lines.

possible with regard to whether the changes in intensity are caused by variations in the ionization conditions.

No special work was done on the classification of the lines into groups. It was observed, however, that the lines measured for intensity gave results that were in excellent agreement with those of C. J. Anger[1] and A. Markov.[2]

This study was made at the suggestion of Dr. Otto Struve and was carried out under his guidance.

YERKES OBSERVATORY
September 1931

[1] *Astrophysical Journal*, **70**, 114, 1928.

[2] *Ibid.*, **72**, 301, 1930.

REVIEWS

Annual Tables of Constants and Numerical Data. Issued by the PERMANENT INTERNATIONAL COMMITTEE (C. MARIE, general secretary). Volume **6,** Part II (1923–1924); Volume **7,** Parts I and II (1925–1926). Paris: Gauthier-Villar; New York: McGraw-Hill Book Co., Inc. 4to. Pp. 681–1684; also 1–946 and 947–1896. $25 per volume bound. Index, Vols. **1–5** (1930). Pp. 382. $16.

Données numériques de spectroscopie. Reprinted from Volumes **5, 6,** and **7** of the foregoing and separately bound. Fr. 125 (pp. 350), Fr. 180 (pp. 363), and Fr. 190 (pp. 514), respectively.

A review of Volume **5** and Part I of Volume **6** of this important encyclopedic publication appeared in this *Journal* in 1929 in the January number. Part II of Volume **6** and Parts I and II of Volume **7** follow the general course of the previous volumes. Biology has been added to the lists of subjects dealt with, and libraries having the other volumes will certainly wish to add these to their shelves. The fields of chemistry, physics, and engineering are also covered.

There has now appeared the comprehensive Index to Volumes **1–5** which in itself forms a considerable volume. More than twenty thousand different substances are classified in this Index and their chemical formulae are given with the list arranged in alphabetical order. An analytical Subject Index in four languages is included, so that ready reference may be made dealing with particular researches. We understand that it is hoped that these volumes will be completed to the number of ten, carrying research through 1930 and together with a second General Index for Volumes **6–10.**

In the previous review the differentiation was shown between these volumes and the *International Critical Tables* published under the auspices of the National Academy of Sciences and the Research Council. Volumes **6** and **7** of the *Critical Tables* have appeared in 1930.

Readers of this *Journal* will find an especial interest in the *Numerical Tables of Spectroscopy* reprinted from the last three volumes of the *Annual Tables.* They are edited by L. Brüninghaus. These are themselves volumes of several hundred pages, as indicated precisely above, and cover

the following principal topics: "Emission Spectra," "Spectroscopic Series," and "Absorption Spectra." Under the heading "Emission Spectra" are given data not only for the elements, but for a great many compounds. The absorption spectra deal with inorganic solids, liquids, and solutions. Many diagrams are reproduced, illustrating these spectra.

The topography retains the good quality of the previous volumes and we are sure that great pains have been taken to avoid errors.

F.

Physikalisches Handwörterbuch. Edited by ARNOLD BERLINER and KARL SCHEEL. 2d ed. Berlin: Julius Springer, 1932. Pp. vi+1428; Figs. 1114. Bound, RM. 99.60; unbound, RM. 96.00.

The first edition of this encyclopedia of the physical sciences appeared in 1924 and contained about eight hundred printed pages. The second edition, which is just off the press, is a volume of more than fourteen hundred pages, containing a useful collection of short articles, arranged in alphabetical order. These articles are the work of ninety-three collaborators representing the various physical sciences, and many of them are excellent reviews of recent work in physics. The book is intended primarily to cover all phases of pure physics; astronomy and astrophysics are treated in so far as they may be of interest to the physicist. Among the contributors of astronomical articles are K. F. Bottlinger (Neu-Babelsberg), E. Freundlich (Potsdam), W. Kruse (Bergedorf), E. Pavel (Potsdam), A. Prey (Prague). The book may be highly recommended to all workers in the physical sciences.

O. STRUVE